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SUPER ELEMENT TECHNIQUE FOR SOLAR ENERGY OPTIMIZATION AT URBAN LEVEL

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Abstract: *The objective of this paper is twofold: first to present a new application of optimization in the field of urban solar analysis and, secondly, to explain a new technique able to solve problems of solar radiation involving hundred thousand degrees of freedom.*

The objective of a sustainable city requires both densification and energy consumption control. Since the industrial period, some efforts have been made to improve the management of the cities, and because the ranges of materials and building techniques were not so wide, the first city decision makers emphasized the search on urban shape, more efficient for the solar energy distribution. It was the single parameter that seemed decisive both for the population comfort (natural light) and for a better thermal efficiency of the buildings (solar contribution). Thanks to the development of numerical simulation methods we want to show that today, it becomes possible to model a full city and to optimize some aspects of its design.

1 INTRODUCTION

Modelling a whole city is complex, because the problem is intrinsically multi-scale and multi-physic. Moreover, different problems can be examined, as daylight availability, heating and cooling demand, or urban climate. Due to the increasing size of the models and the need to develop tools and to perform analyses in an integrated environment, the finite element method is an ideal frame, because it benefits of many years of experience and development.

The first part of this paper is dedicated to explain how to integrate the radiosity method in the finite element context.

In the second part, we show the basic behavior of solar radiation in the particular urban context. We conclude that very few reflections are necessary to obtain a sufficient precision. The concept of envelopes allows decreasing the geometric complexity of the model and introduces to new ideas for improving the performances of the solver.

This need of lowering the geometrical complexity is then related to the classical technique of super element used since the beginning of the finite element method development [1]. Recording that the super element technique can be viewed as a matrix condensation technique, we will examine the characteristics of the view factors matrix, which gives the base of all the subsequent operations, like the construction of the radiosity matrix and the solution of the thermal radiative equations. This technique matches geometry and matrix properties of the radiosity problem in order to decrease the size and to improve the storage capacities.

Finally, it can help to implement optimization techniques of solar radiation at the city level, based not only on genetic algorithms, but also on gradient methods.

2 RADIOSITY EQUATION

The *form factor* (also called *view factor*) is the basic ingredient of radiative heat transfer studies [2, 3]. It defines the fraction of the total power leaving patch A_i that is received by patch A_j. Its definition is purely geometric. The angles θ_i and θ_j relate to the direction of the vector connecting the differential elements with the vectors normal to these elements; r is the distance between the differential elements.

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} v(y_i, y_j) dA_i dA_j \tag{1}$$

Except in particular situations, it is not possible to compute the view factors explicitly [4]. An additional difficulty appears in presence of obstructions represented in the above expression by the visibility function $v\ (y_i, y_j)$. This function is equal to 0 or 1 according to the possible presence of an obstacle that does not allow seeing an element y_i from an element y_j.

It is much easier to compute the differential form factor by removing the external integration that will be taken into account only in a second step to achieve the evaluation of the form factor, using, for instance, Gaussian quadrature rule. The differential form factor in a point surrounded by the element of area dS is given by:

$$F_{dS-A_j} = \int_{A_j} \frac{\cos\theta_i\ \cos\theta_j}{\pi r^2} v(y_i, y_j)\ dA_j \tag{2}$$

In order to solve efficiently the interaction problem, it is usual to set up a discrete formulation derived from the global illumination equation by making the following assumption. The environment is a collection of a finite number N of small diffusively reflecting patches each one, with uniform radiosity [3].

Let us define R, the diagonal matrix containing the hemispherical diffuse reflectances.

$$R_{ij} = \rho_i \delta_{ij} \tag{3}$$

Let denote F the matrix of form factors coefficients between patches i and j as computed in (1):

$$F = \begin{pmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & & \vdots \\ \vdots & & & \vdots \\ F_{N1} & \cdots & \cdots & F_{NN} \end{pmatrix} \tag{4}$$

When the patches are planar polygons, the terms F_{ii} are equal to zero. These coefficients also verify the closure property when the whole environment, scene and sky, is taken into account:

$$\sum_{i=1}^{N} F_{ij} = 1 \quad ; \quad i = 1, N \tag{5}$$

In the next formula, the variables B_i are the radiosities, or radiant fluxes per unit area, on patches i and E_i are the radiant exitances. The radiosity equations can be written:

$$(I - RF)B = E = MB \tag{6}$$

This discrete formulation leads to a linear system of equations for which many algorithms are available. The RF matrix, formed by the products of the form factors by the reflectances, is a non symmetric matrix (except if all the reflectances and patch areas are equal), but the radiosity matrix M is diagonally dominant and well conditioned.

3 SYMMETRIC RADIOSITY EQUATIONS

In order to integrate the radiosity method in the environment of finite element method [5], it is suitable to work with symmetric matrices.

The equation structure allows introducing an important property of the radiative exchanges: the principle or reciprocity

$$\forall (i,j) \ : \ A_i F_{ij} = A_j F_{ji} \tag{7}$$

We rewrite (6) explicitly and divide each line i by A_i/ρ_i

$$\frac{A_i}{\rho_i} B_i - A_i \sum_{k=1}^{n} B_k F_{ik} = \frac{A_i}{\rho_i} E_i \tag{8}$$

In pure diffuse reflection, this relation expresses the energy transfers between the N elements of the scene. If we use the reciprocity relation, we can transform (6) by multiplying the form factor matrix F by the diagonal matrix $S_{ij} = A_i \ \delta_{ij}$ of the patch areas. We obtain then a symmetric matrix with $N(N+1)/2$ elements.

$$SF = \begin{bmatrix} 0 & A_1F_{12} & A_1F_{13} & \cdots \\ A_2F_{21} & 0 & A_2F_{23} & \cdots \\ A_3F_{31} & A_3F_{32} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{9}$$

Then, multiplying (6) by SR^{-1}, we write:

$$\left(SR^{-1} - SF\right)B = SR^{-1}E \tag{10}$$

And in symmetrical form:

$$S\left(R^{-1} - F\right)B = SR^{-1}E \quad \rightarrow \quad B = \left(R^{-1} - F\right)^{-1} R^{-1}E \tag{11}$$

The second member of the first relation represents the incident power on the patch [6]. To solve this system of linear equations, a lot of very efficient methods are available. The Cholesky [7] one is very well known in the field of finite element method. We have good feedback of it for problems with more than one million of degrees of freedom. For hundreds of degrees of freedom, it works very well on PCs.

In each line i of matrices F or SF, the nonzero terms indicate what elements are visible from element i. So, we can build an incidence matrix L composed of integers, which gives the connections between all the elements of the scene. It will help us to manage the system of equations and to identify possible super elements.

Despite the fact that the heaviest part of the computation time is the evaluation of matrix F, we can also try to accelerate the step of solution by using iterative methods as explained in the next section.

4 NEUMANN SERIES

If $G = RF$ (6) has a norm less than one, the matrix M is invertible and the Neumann series of successive multiplications of G will converge to the inverse.

$$if \ \|G\| < 1 \ then \ M^{-1} = \left[I - G\right]^{-1} = \sum_{a=0}^{\infty} G^a \tag{12}$$

This property gives indications to develop very efficient methods to solve these equations. It also gives justifications for iterative solutions. As noted by several authors [1, 8, 9, 10], each step of the iterative process can be interpreted as the introduction of an additional reflection on all the elements of the scene.

Figure 1: One reflection

Figure 2: Two reflections

The ability to decompose the solution of the radiosity equation in *orders of reflection* is very interesting, because it allows comparing this method with the ray tracing one, where the order of reflections is a usual stopping criterion. Thus, the calculation is often stopped at the second reflection. This is true in ray trace software as Radiance [11], but it is also the case for a radiosity solver like V-Ray software [12]. In the latter, we can choose one or two reflections.

In a city, multiple reflections are possible, for instance between facades of narrow streets. Considering an average reflectance of 20%, the energy flow is no more than 20% after the first reflection, 4% after the second one and less than 1% after the third one. However, if someone is interested in local results, the overall reasoning can be confusing, because the reflected energy may be the only available on certain surfaces, where it takes a considerable importance.

In an inner space, the radiation from the Sun and the sky through the window illuminates largely the floor and part of the walls, but it leaves the ceiling in full shade. The first reflection on the ground is the one that illuminates the ceiling. As it is generally light in color, the ceiling returns a second non-negligible reflection to the ground. This light is the first to reach parts of the ground from where the sky is not visible. Two reflections are therefore needed to get a realistic rendering of an interior space in natural light.

But what happens in an outdoor scene? Two illustrations show an urban courtyard illuminated by the Sun and the sky with one reflection (Figure 1) and two reflections (Figure 2). Of course, we observe differences, but the structure of the image (shadows shape, order of increasing brightness, main gradations) is not changed. In an urban scene, because we can almost always see a bit of the sky, the second reflection does not represent a substantial change in the results, and the following ones can be ignored (except in very specific configurations, as for example the entrance of a tunnel).

Modern cities all share some essential characteristics: a network of streets delineates parcels built with heights ranging from a few meters to tens of meters. However, other features are highly variable. This is the case of the coatings optical properties. Facades can be dark (brick) or light (limed walls), with a rate of glazing (and so, specular reflection) from few percent to almost 100% (towers of glass and steel). In the example shown here, an area with very clear facades (reflection coefficient equal to 78%) with dark roofs (22%) has been considered.

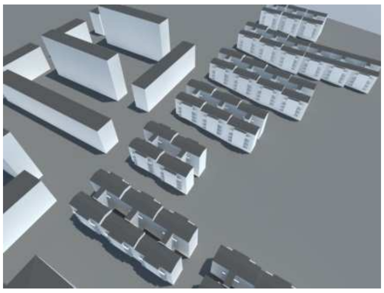
Figure 3: Sun height 50°

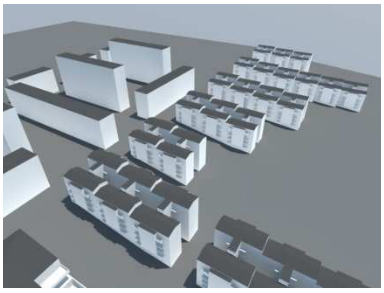
Figure 4: Sun height 35°

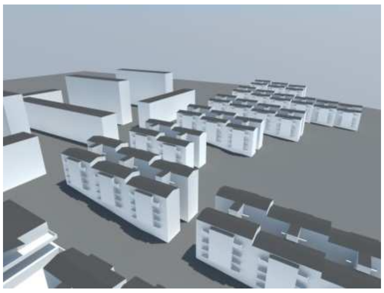
Figure 5: Sun height 20°

An important parameter of environmental physics is the *albedo*. This is an average reflection coefficient over a very large area. For instance, we can refer to the albedo of a planet (the Earth albedo is about 30 %, [13]). The albedo of sea ice, ocean, desert or forest is fairly easy to assess. Today, while cities cover large parts of the land area, it is necessary to know their albedo. However, the semi-regular structure of cities gives highly variable albedo. In our example, viewed from above (Figure 3), the city has the darkest areas of its roofs, but when we move down toward the horizon, we see mostly clear surfaces (Figure 5). The relationship between apparently light and dark surfaces also depends on building height and density of the neighborhood.

Another characteristic of urban settings, due to the fact that cities are relatively low and very spread out, is that what we can see from a given point is very variable. From a window on a ground floor, the view can be limited to only two surfaces: the street and the facing wall. From a window at the top of a tower, we can see dozens, even hundreds of buildings, as in previous images. Calculating an urban geometry therefore strongly motivates to play on the buildings level of detail.

In Figure 7, the distant buildings have been replaced by their prismatic envelopes. This kind of procedure has been used for a long time to accelerate the detection of visible surface. Several options are available; since bounding boxes [14] to prismatic envelopes and convex bounding polyhedra.

Figure 6: Urban parcel

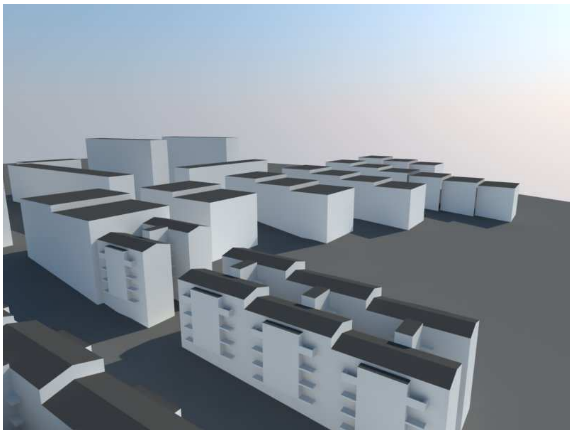

Figure 7: Using envelopes

5 SUPER ELEMENT TECHNIQUE

The motivation for using super element technique is to save computation time and storage capabilities [15]. Their definition is related to the solution of a linear system. It can be seen as a method of matrix condensation.

$$\begin{bmatrix} M_{rr} & M_{rc} \\ M_{cr} & M_{cc} \end{bmatrix} \begin{bmatrix} X_r \\ X_c \end{bmatrix} = \begin{bmatrix} Y_r \\ Y_c \end{bmatrix} \tag{13}$$

The variables are split in two groups: the first one with index r saved for the next steps and the second one with index c that is condensed or temporarily eliminated. The reduced system is:

$$X_c = M_{cc}^{-1}\left(Y_c - M_{cr}X_r\right) \tag{14}$$

Assuming that the variables X_r have been evaluated before, we obtain X_c by solving the second line of (13):

$$(M_{rr} + M_{rc}M_{cc}^{-1}M_{cr})X_r = Y_r - M_{rc}M_{cc}^{-1}Y_c \tag{15}$$

The condensed system is:

$$M_{rr}^{*}X_r^{*} = Y_r^{*} \tag{16}$$

The generation of a super element is performed by selecting a group of elements assumed to be strongly connected. This operation is performed by using the connection or incidence matrix L. The process is the following: after selecting randomly a first element, we repeat the operation for all the elements of the same line. All the elements that have a sufficient connection, i.e., a sufficient number of elements sharing the same links, are candidates to form the super element. At the end, the elements that do not have relation with elements outside this group or that have few connections outside are condensed like in (16). When the definition of the first super element is fixed, we start with a new one until all the elements have been examined or accepted inside a super element.

This process can be fully automated and is similar to the processes of bandwidth and front width optimization in finite element method [16, 17].

This process allows an efficient storage of the matrices and gives the possibility to pre-compute part of the solution. It is also a good method to optimize the solution itself, in the sense that it can provide the optimal sequence of variables in the solution process.

6 MANAGEMENT OF THE OPTIMIZATION

To perform geometric optimization of a set of building, an urban district, etc., several methods are available. In the frame of city optimization, some experience is yet available [18, 19], using generally genetic algorithms. The discussion about the most relevant optimization method is still open [20, 21]. Here, we discuss the alternative of gradient methods that looks very suitable for massive applications.

The goal is to limit the number of iterations (few tens). However, at each step, it is necessary to compute the sensitivities with respect to the variations of the design variables. That implies several extra analyses on very similar geometric models. To be efficient, the model has to be modified very easily. It can be achieved thanks to the incidence matrix, which allows better management of the sub matrices and consequently leads to faster solution and improved storage of the matrices.

The incidence matrix is describing the relations between all the elements defining the scene. When the view factor of an element related to another one is zero, it means that it is not seen from it (or not connected to it). By definition, the sum of the solid angles of all the elements (including the sky ones) surrounding the differential element of a flat surface is equal to 2π and, with the same condition, the sum of view factors is equal to π. Consequently, summarizing the normalized elements of a line of the solid angle factors or of the view factors matrix gives always one.

These properties, combined with the geometrical description of the scene: localizations of the elements, vicinity relations etc…, allow identifying sub matrices that will not be modified during the optimization process and consequently to pre-compute them and to perform some of the matrix operations only one time.

These matrix evaluations are also giving information on the coupling of different sub-scenes like districts or streets. If the optimization is, for instance, limited to a district, this knowledge leads to efficient strategies for evaluating the objective function and the different derivatives necessary to set up gradient methods.

This kind of improvement was obtained in the thermal analysis of spacecrafts [22] in which it is mandatory to control very precisely the temperatures of the components and to favor the thermal fluxes. It could be extended to the optimization of urban districts, but it is still necessary to specify the design variables, the relevant constraints and, finally, the objective functions of such a problem.

7 CONCLUSIONS

The proposed acceleration techniques of the solution allow using, in the optimization step, gradient techniques instead of genetic like algorithms that are difficult to use in problems involving hundred thousands of degrees of freedom.

As a conclusion, grouping the elements in super elements with adequate criteria allows a better use of the coupling terms in the matrices of the equations systems and the saving of part of the computations to accelerate the optimization process.

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