

# Complete set of Matlab procedures for achieving uniform ray generation

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## 1. Introduction

The goal of this work is to provide efficient and accessible Matlab functions for anyone using ray tracing. The proposed methods and functions are considered as the most suitable ones to achieve uniform rays distribution. The foundations of these developments are explained in details in [Beckers & Beckers, 2014]. The figures of this document are giving both a Matlab statement in the header and the related examples below. The tables are displaying complete functions that can be used by a simple “copy paste” operation with the name shown in the table header. The red headers correspond to the equal solid angles technique (*ESA*), while the blue headers refer to the equal view factors (*EVF*) method.

## 2. Equal solid angle cells generation on the hemisphere

The first illustration consists in generating an equal area or equal solid angle (*ESA*) mesh on the hemisphere. The cells, except the polar one, are defined between two longitudes and two latitudes. In *Figure 1*, we see a dome composed of 145 cells. To generate it, we have just to call the function *Bsamd* (*Table 1*) with a single input: the number of requested cells (here 145) and a single output: the array *S*, which contains the cumulated number of cells in the nested spherical caps:  $S = [1\ 8\ 20\ 38\ 60\ 86\ 115\ 145]$  (*Figure 1*, bottom right).

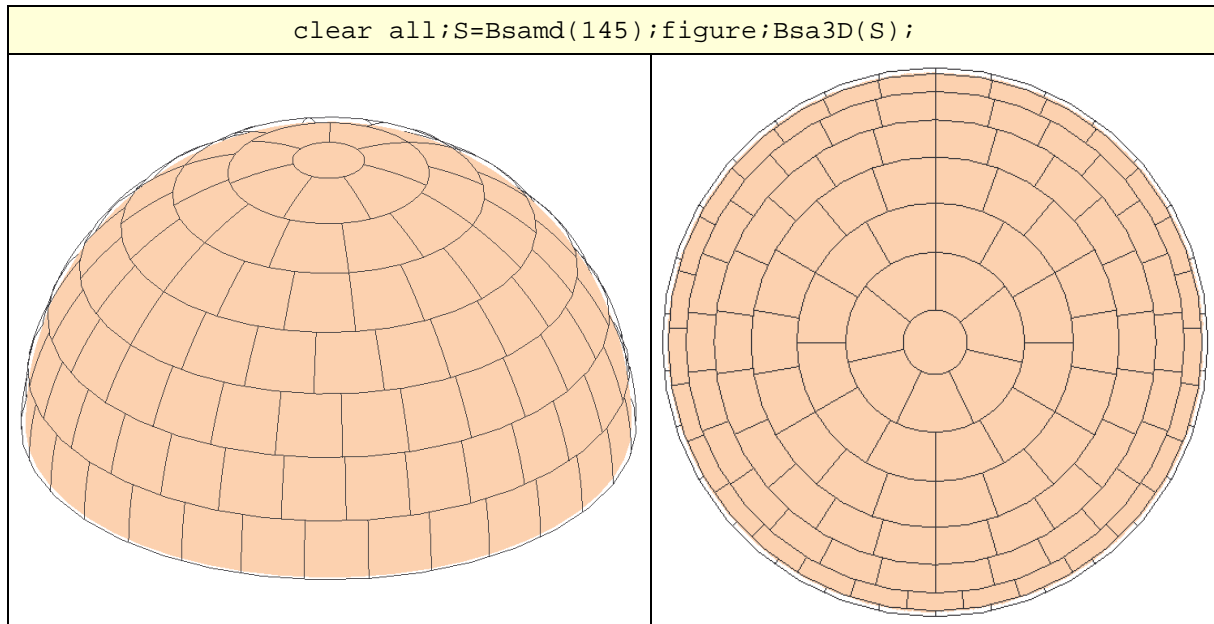


Figure 1: Generation and representation of a 145 **ESA** cells dome, using **Bsamd** & **Bsa3D** functions

The statement of [Figure 1](#) is also calling another function: [Bsa3D](#) ([Table 4](#)) which is producing the drawing, its input is the output of the function [Bsamd](#).

Bsamd (solid angle mesh dome)	
1	<code>function [S] = Bsamd(idep)</code>
2	<code>if nargin == 0, idep = 145; end</code> <span style="float: right;">% Default value of the input</span>
3	<code>Ims = 1;ccp=7;</code> <span style="float: right;">% Tuning parameters</span>
4	<code>nring = floor(sqrt(idep));</code> <span style="float: right;">% Defining a tentative realistic number of rings</span>
5	<code>tp = ones(1,nring)*pi/2;Sp=ones(1,nring)*idep;nan = 0;% initializations</code>
6	<code>for i = 2:nring;</code>
7	<code>tp(i) = tp(i-1)-sqrt(2*pi/idep);</code>
8	<code>Sp(i) = round(Sp(i-1)*(sin(tp(i)/2)/sin(tp(i-1)/2))^2);</code>
9	<code>if Ims &gt; 0;tp(i)=2*asin( sin(tp(i-1)/2)*sqrt(Sp(i)/Sp(i-1)));end;</code>
10	<code>if Sp(i-1)&lt;ccp;Sp(i-1)=1;Sp(i)=0;end% Forcing presence of central disk</code>
11	<code>if Sp(i-1) == 1;if nan == 0;nan = i-1;end; end</code>
12	<code>end</code>
13	<code>S=zeros(1,nan);for i=nan:-1:1;S(nan-i+1)=Sp(i);end; % Re-ordering the caps</code>
14	<code>if size(S,2) &lt; 11;</code>
15	<code>disp(['SA Sequence of ',num2str(size(S,2)), ' nested caps: ',num2str(S)])</code>
16	<code>end</code>
17	<code>end</code>

Table 1: Function **Bsamd**. Equal solid angle cells sequence generation on the hemisphere (**ESA**)

### 3. Equal view factor cells generation on the hemisphere

Instead of generating equal area cells, we can also want to produce equal view factor cells (**EVF**). As before, we can do it with the statement of [Figure 2](#) header. We observe that this dome is different from the **ESA** one of [Figure 1](#). According to the theory, its orthogonal projection in the base plane contains equal area cells (Nusselt analogy, [[Beckers & Beckers, 2014](#)]), what is not the situation in the [Figure 1](#) case. Like in the first example, this statement is using two functions: [Bvfm](#) to generate the sequence of cumulated cells  $S = [1 \ 12 \ 28 \ 49 \ 74 \ 101 \ 127 \ 14]$  and the function [Bvf3D](#) ([Table 10](#)), which is producing the drawing; its input is the output of the function [Bvfm](#).

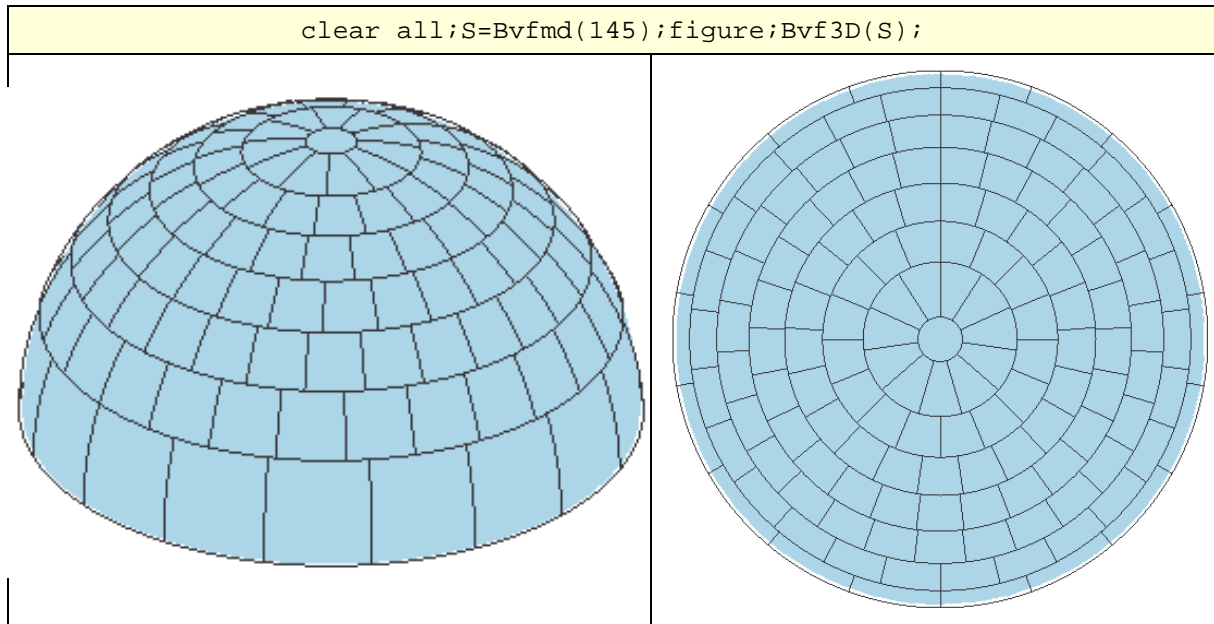


Figure 2: Generation and representation of a 145 EVF cells dome using **Bvfmfmd** & **Bvf3D** functions

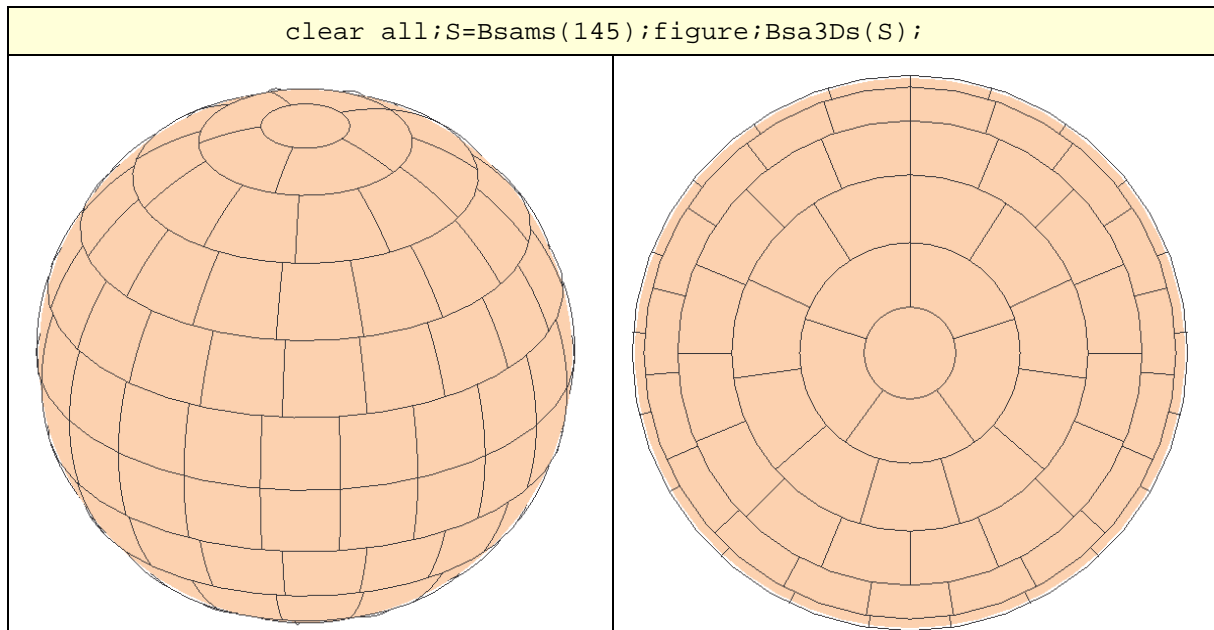
<b>Bvfmfmd (view factor mesh dome)<sup>1</sup></b>	
1	<code>function [S] = Bvfmfmd(idep)</code>
2	<code>if nargin == 0, idep = 145; end</code> <span style="float: right;">% Default value of the input</span>
3	<code>Ad = pi/idep;</code> <span style="float: right;">% Ad = disk cell area</span>
4	<code>nring = floor(sqrt(idep));</code> <span style="float: right;">% Defining a tentative realistic number of rings</span>
5	<code>tp = ones(1,nring)*pi/2; Sp=ones(1,nring)*idep;</code> <span style="float: right;">% initializations</span>
6	<code>r = ones(1,nring); ncr=round(idep/nring);</code> <span style="float: right;">% initializations</span>
7	<code>Sp(2) = Sp(1)-ncr; nan=0;</code> <span style="float: right;">% initializations</span>
8	<code>for i = 2:nring;</code>
9	<code>if Sp(i-1) &gt; 1;</code>
10	<code>Sp(i) = Sp(i-1)-10;</code>
11	<code>ar = 10;</code>
12	<code>while ar &gt; 1;</code>
13	<code>arp = ar;</code> <span style="float: right;">% Possible optimization Stat. 1</span>
14	<code>Sp(i) = Sp(i)-1;</code>
15	<code>ncr = Sp(i-1)-Sp(i);</code>
16	<code>r(i) = sqrt(r(i-1)^2-Ad*ncr/pi);</code>
17	<code>tp(i) = asin(r(i));</code>
18	<code>acell = 2*pi*(cos(tp(i))-cos(tp(i-1)))/ncr;</code>
19	<code>ar = acell/(tp(i-1)-tp(i))^2;</code>
20	<code>end</code>
21	<code>% if arp-1 &lt; 1-ar; Sp(i)=Sp(i)+1; end</code> <span style="float: right;">% Possible optimization Stat. 2</span>
22	<code>if Sp(i-1)&lt;6; Sp(i-1)=1; Sp(i)=1; end</code> <span style="float: right;">% Forcing presence central disk</span>
23	<code>if Sp(i-1) == 1; if nan == 0; nan = i-1; end; end;</code>
24	<code>else</code>
25	<code>nan=i-1;</code>
26	<code>end</code>
27	<code>end</code>
28	<code>if Sp(nan) == Sp(nan-1); nan = nan-1; end</code>
29	<code>S=zeros(1,nan); for i = nan:-1:1; S(nan-i+1)=Sp(i); end;</code> <span style="float: right;">% Re-ordering caps</span>
30	<code>if S(1) &lt;= 0; S(1)=1; end</code>
31	<code>if nan &lt; 11;</code>
32	<code>disp(['VF Sequence of ', num2str(size(S,2)), ' nested caps: ', num2str(S)])</code>
33	<code>end</code>
34	<code>end</code>

Table 2: Function **Bvfmfmd**. Equal view factor cells sequence generation on the hemisphere (EVF)

<sup>1</sup> Procedure modified in November 2016 to remove an useless argument.

## 4. Equal solid angle cells generation on the sphere

In the case of *Figure 3*, we are following the same scheme as before, but with two new functions *Bsams* and *Bsa3Ds* given in *Table 3* & *Table 7*.



*Figure 3: Generation and representation of a 145 ESA cells sphere, with Bsams & Bsa3Ds functions*

<b>Bsams (solid angle mesh sphere)</b>	
1	<code>function [S] = Bsams(nsph)</code>
2	<code>if nargin == 0;nsph = 290;end; % Input default value</code>
3	<code>if nsph &lt; 8;nsph=8;end</code>
4	<code>nsph = round(nsph/2)*2; % Number of cells must be even</code>
5	<code>idep = nsph/2;timl=pi/2;riml=sqrt(2);niml=idep; % Initializations</code>
6	<code>nring = floor(sqrt(idep)); % Estimated number of rings</code>
7	<code>n = zeros(1,nring);nan = 0; % initializations</code>
8	<code>for i = 1:nring; % Loop on the rings or disks</code>
9	<code>    n(i) = niml; % Number of cells in disk i</code>
10	<code>    ti = timl-sqrt(2*pi/idep); % Zenithal angle (20)</code>
11	<code>    ri = 2*sin(ti/2); % equivalent projection (16)</code>
12	<code>    ni = round(niml*(ri/riml)^2); % Number of cells (1)</code>
13	<code>    niml = ni;riml = ri;timl = ti;</code>
14	<code>    if niml == 2;niml = 1;end % Forcing presence of polar disks</code>
15	<code>    if niml == 0;niml = 1;end</code>
16	<code>    if niml == 1;if nan == 0;nan = i+1;end;end</code>
17	<code>end</code>
18	<code>S = [n(nan:-1:1) 2*n(1)-n(2:nan) nsph];</code>
19	<code>if size(S,2) &lt; 13;</code>
20	<code>    disp(['SA Sphere sequence of ',num2str(size(S,2)),' layers: ',num2str(S)])</code>
21	<code>end</code>
22	<code>end</code>

*Table 3: Function Bsams. Equal solid angle cells sequence generation on the sphere (ESA)*

## 5. Ray generation

When we benefit from a uniform mesh created on the sphere or the hemisphere, we can easily define rays, either from particular positions in the elements, for instance the barycenter (deterministic rays) or from random points created inside the elements. For the second situation, we take advantage of the quadrangular shape of the elements. It is especially simple to generate random coordinates defined between two latitudes and two longitudes.

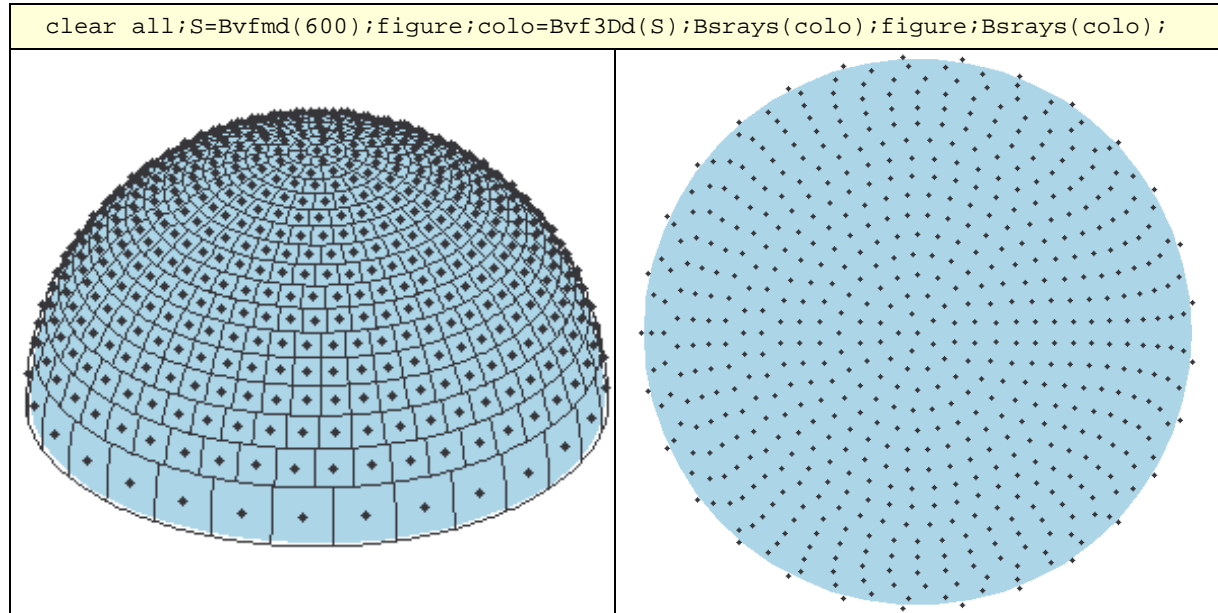


Figure 4: Generation of 600 *EVF* cells & deterministic rays, with *Bvfmd*, *Bvf3Dd* & *Bsprays*

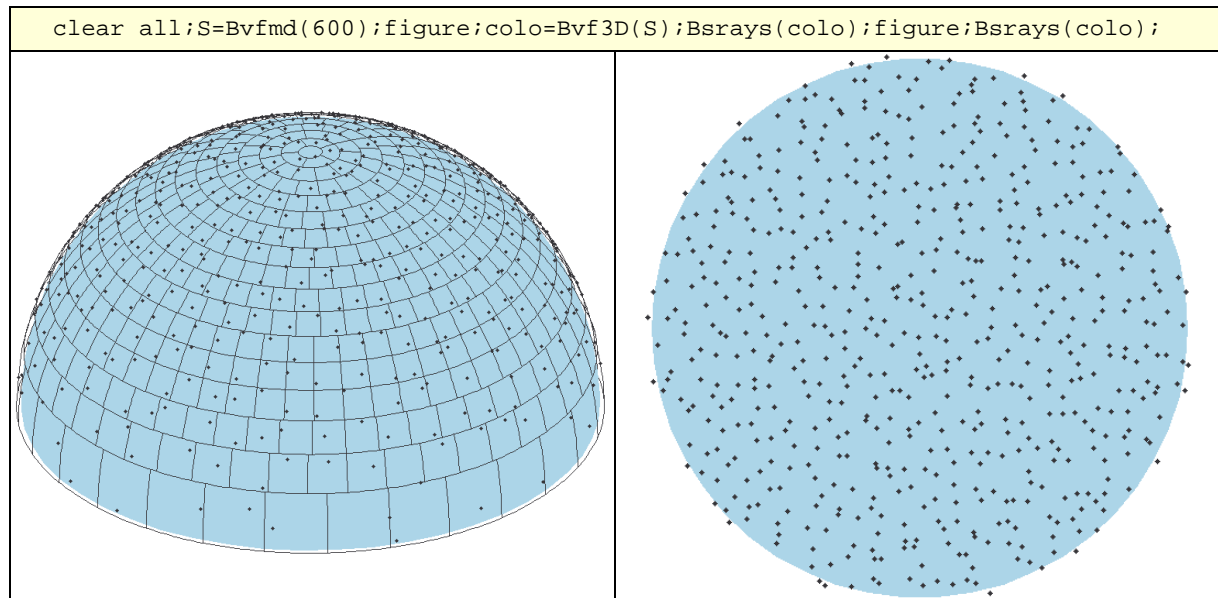


Figure 5: Generation of 600 *EVF* cells and random rays, with *Bvfmd*, *Bvf3D* & *Bsprays* functions

The left part of *Figure 6* shows an *ESA* mesh of the sphere composed of 490 elements. The right part also contains the random points generated in the cells. The statements used to produce both drawing are presented in the header of the figure. In the second one, we observe the instruction `colo = Bsa3Ds(S)` indicating that the spherical coordinates of the points are recovered in the array `colo`. Their representation is obtained through the function *Bsprays* (*Table 16*).

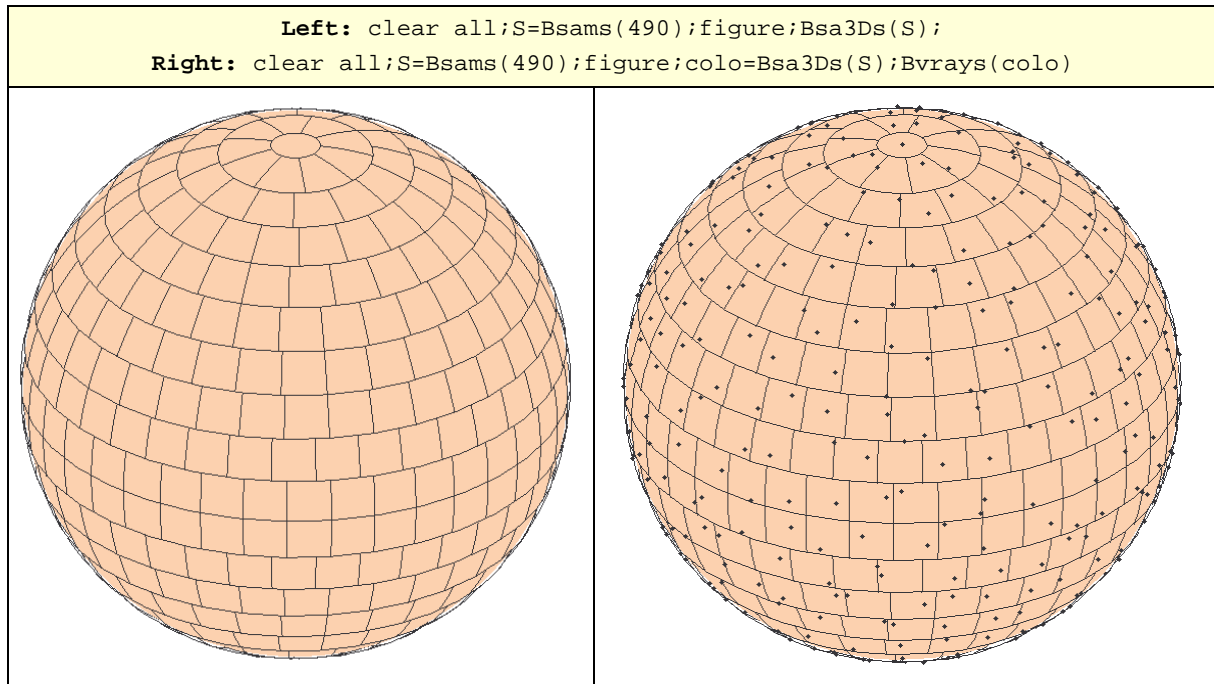


Figure 6: Sphere composed of 490 elements (left) and their 490 superposed random rays (right)

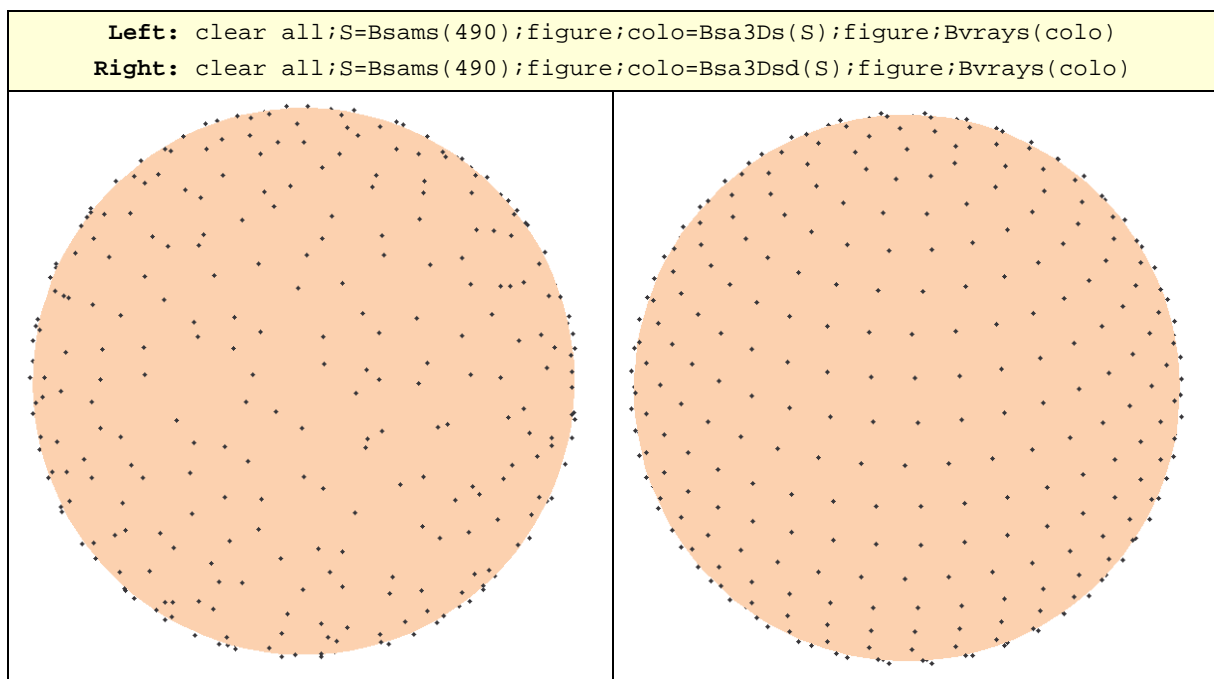


Figure 7: Sphere composed of 490 random rays (left) and 490 deterministic rays (right)

To represent the coordinates of the rays only, it is sufficient, like in [Figure 7](#), to reset the drawing by calling the `figure` Matlab function. In [Figure 7](#), the statements leading to both drawings only differ by calling `Bsa3Ds` ([Table 7](#)) for the random rays or `Bsa3Dsd` ([Table 8](#)) for the deterministic ones.

Evaluating the cost of a ray generation is performed with the following sequence, with the new function `Bsas` ([Table 9](#)), which only computes the spherical coordinates from the sequence  $S$ . In the header, we have also introduced statements to measure the elapsed time between `"tic"` and `"toc"`.

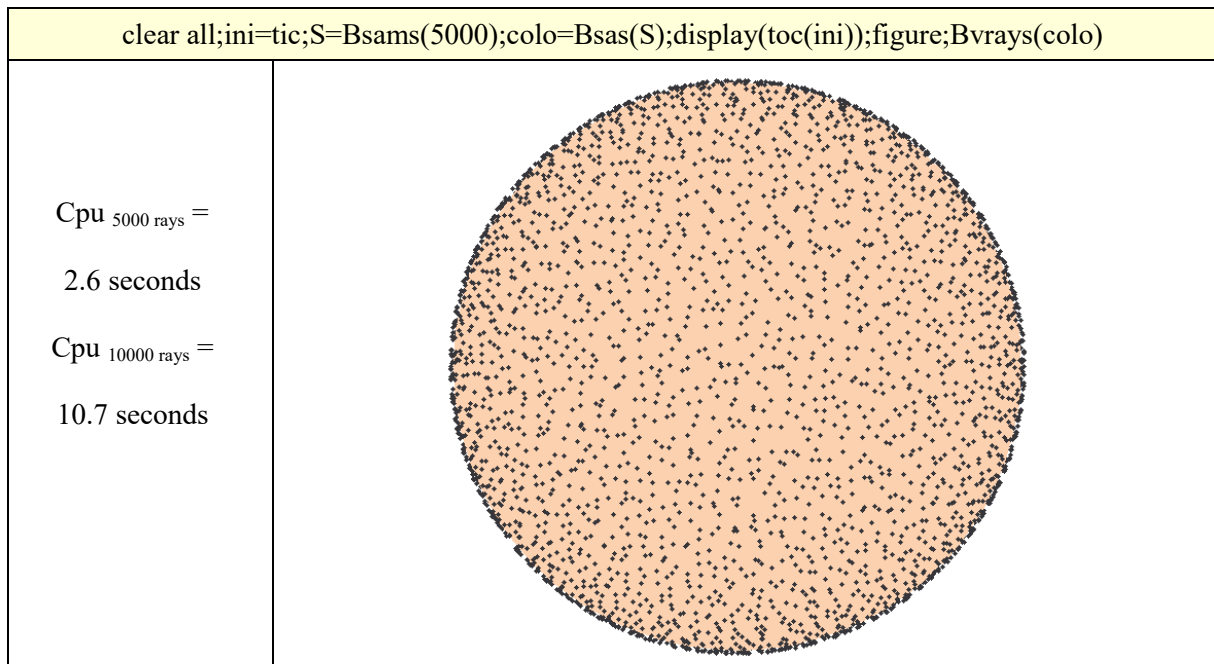


Figure 8: Sphere composed of 5000 *ESA* random rays

We can do the same with the domes by calling functions purely dedicated to the generation of rays. For the view factors, we use *Bvf* (*Table 11*).

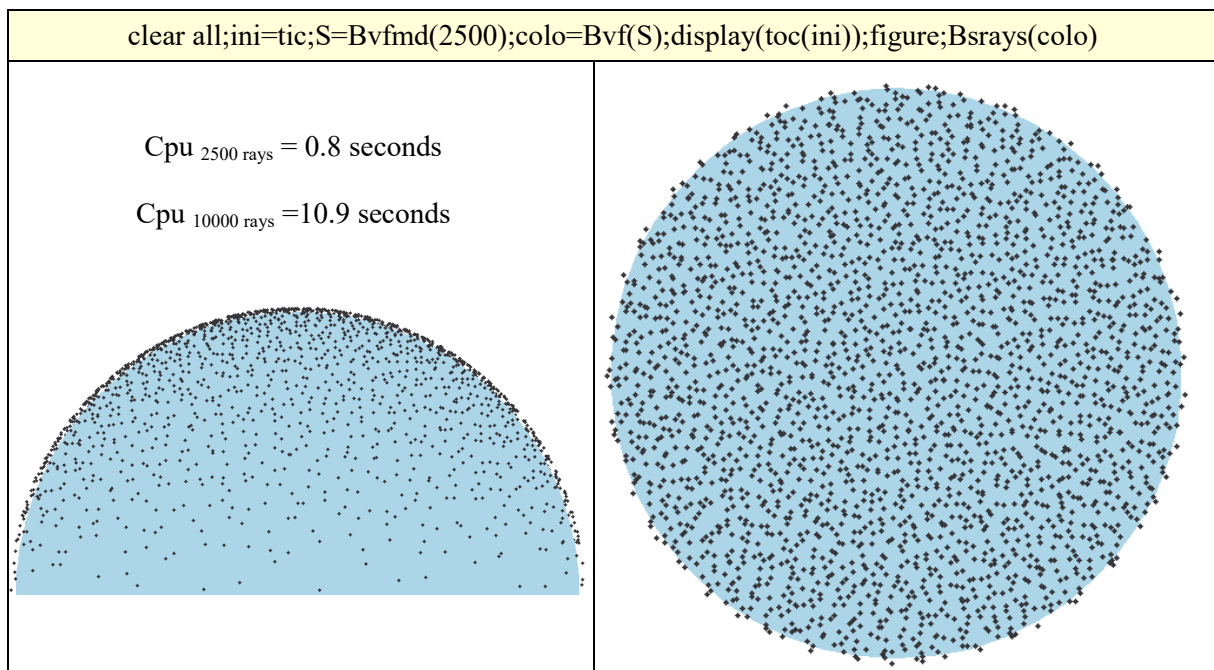


Figure 9: Dome composed of 2500 *EVF* random rays

When comparing *Figure 8* and *Figure 9*, we observe that the uniformly distributed points of the sphere seem to present higher density close to the limits of the disk of *Figure 8* that contains their orthogonal projection, while the density appears to be perfectly uniform in *Figure 9*, which corresponds to the orthogonal projection of a non uniform distribution of points on the hemisphere.

## 6. Conclusion

The procedures given in *Table 1* to *Table 3* are very compact and allow generating meshes composed of cells of equal area and aspect ratio close to 1 (*ESA*) or equal view factor and aspect ratio close to 1 (*EVF*). This set is completed by a set of procedures helping to draw the meshes or to list the coordinates of the rays.

## 7. Matlab procedures

### 7.1. Solid angles: rays generation and 3D drawing

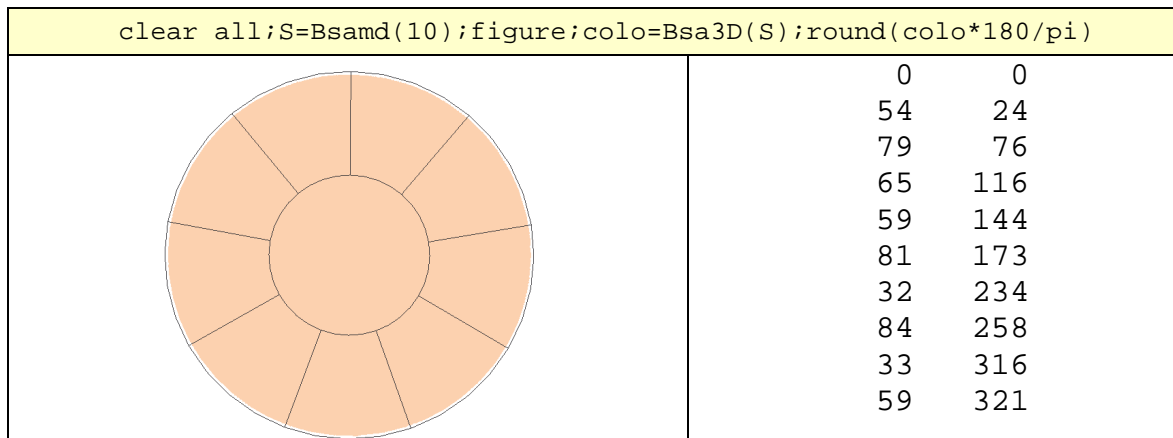
The objective of the function of *Table 4* is to draw meshes defined by any array *S* and to compute the spherical coordinates of the set of random rays generated in the two columns array *drays*, when it is present in the call of the function like in *Figure 10*.

<b>Bsa3D (solid angle 3D dome &amp; random rays)</b>	
1	<code>function [drays] = Bsa3D(S)</code>
2	<code>cs = 20; npas=8; b=(0:10:360)*pi/180; % Parameter for the execution</code>
3	<code>Nc = size(S,2); Nt = S(Nc); R = S(2:Nc)-S(1:Nc-1); % Definition of the rings</code>
4	<code>aleax=rand(Nt); aleay=rand(Nt); % Generation of 2 x Nt random numbers</code>
5	<code>t = acos(1-S/Nt); r = sin(t); h = cos(t); % from disk equal area to 3D</code>
6	<code>for i= 1 : Nc; % Drawing the parallels in axnometric projection</code>
7	<code>    x = r(i)*cos(b); y = r(i)*sin(b); z = ones(size(b,2))*h(i);</code>
8	<code>    plot3(x,y,z,'k'); hold on;</code>
9	<code>end</code>
10	<code>la=zeros(Nt,1); lo=zeros(Nt,1); n=1;</code>
11	<code>for i = 1 : Nc-1; % Drawing the meridians segments</code>
12	<code>    lp = 0.;</code>
13	<code>    for j = 1 : R(i);</code>
14	<code>        longi = j*2*pi/R(i);</code>
15	<code>        lai = t(i); las = t(i+1); pala = (las-lai)/npas;</code>
16	<code>        mxi = sin(lai:pala:las)*cos(longi);</code>
17	<code>        myi = sin(lai:pala:las)*sin(longi);</code>
18	<code>        mzi = cos(lai:pala:las);</code>
19	<code>        plot3(mxi',myi',mzi','k'); hold on</code>
20	<code>        if nargout &gt;0;</code>
21	<code>            n=n+1;</code>
22	<code>            la(n)=t(i)+aleax(n)*(t(i+1)-t(i));</code>
23	<code>            lo(n)=lp +aleay(n)*(longi-lp); lp=longi;</code>
24	<code>        end</code>
25	<code>    end</code>
26	<code>end;</code>
27	<code>if nargout &gt; 0;</code>
28	<code>    drays=[la lo];</code>
29	<code>end</code>
30	<code>title(['Equal solid angle dome composed of ', num2str(Nt), ' elements'],...</code>
31	<code>    'fontsize',cs);</code>
32	<code>Bwdo; hold on; axis equal; axis off;</code>
33	<code>end</code>

Table 4: Function **Bsa3D**. Drawing of the mesh and eventually generation of random rays



Using the procedures of *Table 1* and *Table 4*, the statement of the header of *Figure 10* is giving the 10 couples of spherical coordinates (in degrees) randomly positioned in the cells. This kind of output is perfectly suited to perform ray tracing based on stratified sampling.



*Figure 10: Example of generation, drawing and random set of rays created for stratified sampling*

<b>Bsa (ESA random rays on a dome)</b>	
1	<code>function [drays] = Bsa(S)</code>
2	<code>Nc = size(S,2);Nt = S(Nc);R = S(2:Nc)-S(1:Nc-1);% Definition of the rings</code>
3	<code>aleax=rand(Nt);aley=rand(Nt);                    % Generation of 2 x Nt random numbers</code>
4	<code>t = acos(1-S/Nt);                                    % from disk equal area to 3D</code>
5	<code>la=zeros(Nt,1);lo=zeros(Nt,1);n=1;</code>
6	<code>for i = 1 : Nc-1;                                    % Drawing the meridians segments</code>
7	<code>lp = 0.;</code>
8	<code>for j = 1 : R(i);</code>
9	<code>longi = j*2*pi/R(i);</code>
10	<code>n=n+1;</code>
11	<code>la(n)=t(i)+aleax(n)*(t(i+1)-t(i));</code>
12	<code>lo(n)=lp +aley(n)*(longi-lp);lp=longi;</code>
13	<code>end</code>
14	<code>end;</code>
15	<code>drays=[la lo];</code>
16	<code>end</code>

*Table 5: Function **Bsa**. Generation of **ESA** random rays on a dome*

### Bsa3Dd (solid angle 3D dome & deterministic rays)

```

1 function [drays] = Bsa3Dd(S)
2 cs = 20; npas=8; b=(0:10:360)*pi/180; % Parameter for the execution
3 Nc = size(S,2); Nt = S(Nc); R = S(2:Nc)-S(1:Nc-1); % Definition of the rings
4 t = acos(1-S/Nt); % from disk equal area to 3D
5 r = sin(t);
6 h = cos(t);
7 for i = 1 : Nc; % Drawing the parallels in axnometric projection
8     x = r(i)*cos(b); y = r(i)*sin(b); z = ones(size(b,2))*h(i);
9     plot3(x,y,z,'k'); hold on;
10 end
11 la=zeros(Nt,1); lo=zeros(Nt,1); n=1;
12 for i = 1 : Nc-1; % Drawing the meridians segments
13     lp = 0.;
14     for j = 1 : R(i);
15         longi = j*2*pi/R(i);
16         lai = t(i); las = t(i+1); pala = (las-lai)/npas;
17         mxi = sin(lai:pala:las)*cos(longi);
18         myi = sin(lai:pala:las)*sin(longi);
19         mzi = cos(lai:pala:las);
20         plot3(mxi',myi',mzi','k'); hold on
21         if nargout > 0;
22             n=n+1; la(n)=(t(i+1)+t(i))/2; lo(n)=(longi+lp)/2; lp=longi;
23         end
24     end
25 end;
26 if nargout > 0; drays=[la lo]; end
27 title(['Equal solid angle dome composed of ', num2str(Nt), ' elements'],...
28 'fontSize',cs); Bwdo; hold on; axis equal; axis off;
29 end

```

Table 6: Function **Bsa3Dd**. Drawing of the mesh and eventually generation of deterministic rays

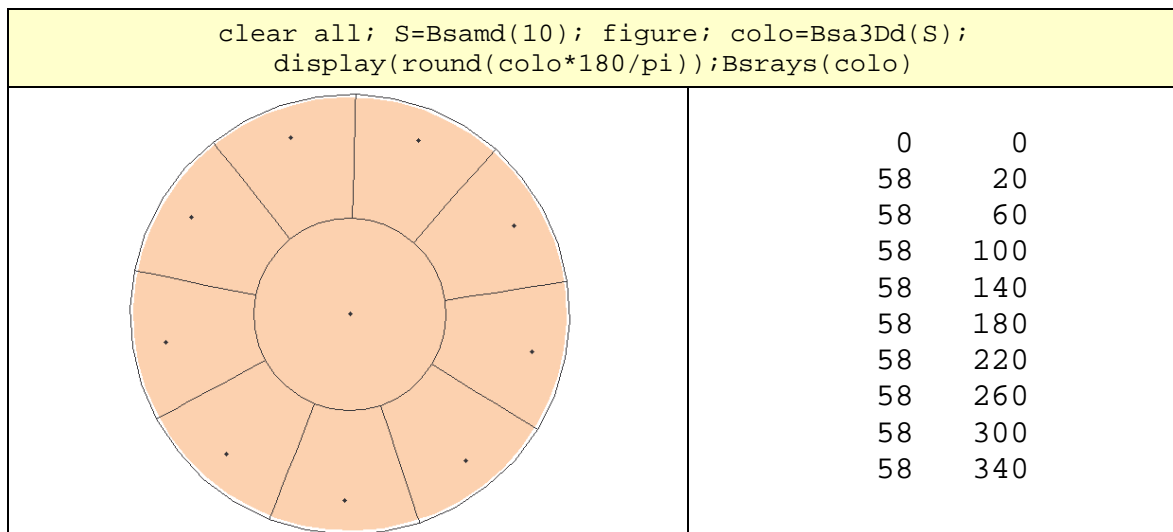


Figure 11: Example of generation of the drawing and deterministic set of rays

### Bsa3Ds (solid angle 3D sphere & random rays)

```

1 function [drays] = Bsa3Ds(S)
2 cs = 20; npas=8; b=(0:10:360)*pi/180; % Parameter for the execution
3 Nc = size(S,2)-1; % Size of the interior points set
4 Nt = max(S); % Number of cells in the hemisphere
5 aleax=rand(Nt); aleay=rand(Nt); % Random vectors
6 R = S(2:Nc)-S(1:Nc-1); % Definition of the rings
7 t = acos(1-2*S/Nt); r = sin(t); h = cos(t); % From disk equal area to 3D
8 for i = 1 : Nc; % Drawing the parallels in axnometric projection
9 x = r(i)*cos(b); y = r(i)*sin(b); z = ones(size(b,2))*h(i);
10 plot3(x,y,z,'k'); hold on;
11 end
12 la=zeros(Nt,1); lo=zeros(Nt,1); n=1;
13 for i = 1 : Nc-1; % Drawing the meridians segments
14 lp = 0.;
15 for j = 1 : R(i);
16 longi = j*2*pi/R(i); lai=t(i); las=t(i+1); pala=(las-lai)/npas;
17 mxi = sin(lai:pala:las)*cos(longi);
18 myi = sin(lai:pala:las)*sin(longi);
19 mzi = cos(lai:pala:las);
20 plot3(mxi,'myi',mzi,'k'); hold on
21 if nargout > 0;
22 n=n+1; la(n)=t(i)+aleax(n)*(t(i+1)-t(i));
23 lo(n)=lp +aleay(n)*(longi-lp); lp=longi;
24 end
25 end
26 end;
27 la(Nt)=pi; lo(Nt)=0; if nargout > 0; drays=[la lo]; end
28 title(['Equal solid angle sphere composed of ', num2str(Nt), ...
29 ' elements'], 'fontsize', cs); Bwba; hold on; axis equal; axis off;
30 end

```

Table 7: Function **Bsa3Ds**. Sphere mesh drawing and eventually generation of random rays

### Bsa3Dsd (solid angle 3D sphere & deterministic rays)

```

1 function [drays] = Bsa3Dsd(S)
2 cs = 20; npas=8; b=(0:10:360)*pi/180; % Parameter for the execution
3 Nc = size(S,2)-1; % Size of the interior points set
4 Nt = max(S); % Number of cells in the hemisphere
5 R = S(2:Nc)-S(1:Nc-1); % Definition of the rings
6 t = acos(1-2*S/Nt); % From disk equal area to 3D
7 r = sin(t); h = cos(t);
8 for i = 1 : Nc; % Drawing the parallels in axnometric projection
9 x = r(i)*cos(b); y = r(i)*sin(b); z = ones(size(b,2))*h(i);
10 plot3(x,y,z,'k'); hold on;
11 end
12 la=zeros(Nt,1); lo=zeros(Nt,1); n=1;
13 for i = 1 : Nc-1; % Drawing the meridians segments
14 lp = 0.;
15 for j = 1 : R(i);
16 longi = j*2*pi/R(i);
17 lai = t(i); las = t(i+1); pala = (las-lai)/npas;
18 mxi = sin(lai:pala:las)*cos(longi);
19 myi = sin(lai:pala:las)*sin(longi);
20 mzi = cos(lai:pala:las);
21 plot3(mxi,'myi',mzi,'k'); hold on
22 if nargout > 0;
23 n=n+1; la(n)=(t(i+1)+t(i))/2; lo(n)=(longi+lp)/2; lp=longi;
24 end
25 end
26 end;
27 la(Nt)=pi; lo(Nt)=0; if nargout > 0; drays=[la lo]; end
28 title(['Equal solid angle sphere composed of ', num2str(Nt), ...
29 ' elements'], 'fontsize', cs); Bwba; hold on; axis equal; axis off;
30 end

```

Table 8: Function **Bsa3Dsd**. Drawing of the mesh and eventually generation of deterministic rays

<b>Bsas (solid angle 3D sphere &amp; random rays)</b>	
1	<code>function [drays] = Bsas(S)</code>
2	<code>Nc = size(S,2)-1;</code> <span style="float: right;">% Size of the interior points set</span>
3	<code>Nt = max(S);</code> <span style="float: right;">% Number of cells in the hemisphere</span>
4	<code>aleax=rand(Nt);aley=rand(Nt);</code> <span style="float: right;">% Random vectors</span>
5	<code>R = S(2:Nc)-S(1:Nc-1);</code> <span style="float: right;">% Definition of the rings</span>
6	<code>t = acos(1-2*S/Nt);</code> <span style="float: right;">% From disk equal area to 3D</span>
7	<code>la=zeros(Nt,1);lo=zeros(Nt,1);n=1;</code>
8	<code>for i = 1 : Nc-1;</code> <span style="float: right;">% Drawing the meridians segments</span>
9	<code>lp = 0.;</code>
10	<code>for j = 1 : R(i);</code>
11	<code>longi = j*2*pi/R(i);</code>
12	<code>n=n+1;la(n)=t(i)+aleax(n)*(t(i+1)-t(i));</code>
13	<code>lo(n)=lp +aley(n)*(longi-lp);lp=longi;</code>
14	<code>end</code>
15	<code>end;</code>
16	<code>la(Nt)=pi;lo(Nt)=0;drays=[la lo];</code>
17	<code>end</code>

Table 9: Function **Bsas**. Generation of random rays on the sphere

## 7.2. View factors: ray generation and 3D representation

<b>Bvf3D (view factor 3D dome &amp; random rays)</b>	
1	<code>function [drays] = Bvf3D(S)</code>
2	<code>cs = 20;npas=8;b=(0:360)*pi/180;</code> <span style="float: right;">% Parameter for the execution</span>
3	<code>Nc = size(S,2);Nt = S(Nc);R = S(2:Nc)-S(1:Nc-1);</code> <span style="float: right;">% Definition of the rings</span>
4	<code>aleax=rand(Nt);aley=rand(Nt);</code>
5	<code>r = sqrt(S/Nt);t = asin(r);h = cos(t);</code>
6	<code>for i= 1 : Nc;</code> <span style="float: right;">% Drawing the parallels in axnometric projection</span>
7	<code>x = r(i)*cos(b);y = r(i)*sin(b);z = ones(size(b,2))*h(i);</code>
8	<code>plot3(x,y,z,'k'); hold on;</code>
9	<code>end</code>
10	<code>la=zeros(Nt,1);lo=zeros(Nt,1);n=1;la(1)=aleax(1)*t(1);lo(1)=aley(1)*2*pi;</code>
11	<code>for i = 1 : Nc-1;</code> <span style="float: right;">% Drawing the meridians segments</span>
12	<code>lp=0.;</code>
13	<code>for j = 1 : R(i);</code>
14	<code>longi = j*2*pi/R(i);</code>
15	<code>lai = t(i);las=t(i+1);pala=(las-lai)/npas;</code>
16	<code>mxl = sin(lai:pala:las)*cos(longi);</code>
17	<code>myl = sin(lai:pala:las)*sin(longi);</code>
18	<code>mzl = cos(lai:pala:las);</code>
19	<code>plot3(mxl,'myl',mzl,'k');hold on</code>
20	<code>if nargout &gt;0;</code>
21	<code>n=n+1;</code>
22	<code>la(n)=t(i)+aleax(n)*(t(i+1)-t(i));</code>
23	<code>lo(n)=lp +aley(n)*(longi-lp);lp=longi;</code>
24	<code>end</code>
25	<code>end</code>
26	<code>end;</code>
27	<code>if nargout &gt; 0;drays=[la lo];end</code>
28	<code>title(['Equal view factor dome composed of ',num2str(Nt),' elements'],...</code>
29	<code>'fontsize',cs);Bwdo(50);hold on;axis equal;axis off;</code>
30	<code>end</code>

Table 10: Function **Bvf3D**. Dome mesh and eventually generation of random rays

<b>Bvf (view factor random rays)</b>	
1	<code>function [drays] = Bvf(S)</code>
2	<code>Nc = size(S,2);Nt = S(Nc);R = S(2:Nc)-S(1:Nc-1);% Definition of the rings</code>
3	<code>aleax=rand(Nt);aley=rand(Nt);</code>
4	<code>r = sqrt(S/Nt);t = asin(r);</code>
5	<code>la=zeros(Nt,1);lo=zeros(Nt,1);n=1;la(1)=aleax(1)*t(1);lo(1)=aley(1)*2*pi;</code>
6	<code>for i = 1 : Nc-1; % Drawing the meridians segments</code>
7	<code>lp=0.;</code>
8	<code>for j = 1 : R(i);</code>
9	<code>longi = j*2*pi/R(i);</code>
10	<code>n=n+1;</code>
11	<code>la(n)=t(i)+aleax(n)*(t(i+1)-t(i));</code>
12	<code>lo(n)=lp +aley(n)*(longi-lp);lp=longi;</code>
13	<code>end</code>
14	<code>end;</code>
15	<code>drays=[la lo];</code>
16	<code>end</code>

Table 11: Function **Bvf**. Generation of random rays on an *EVF* dome

<b>Bvf3Dd (Beckers view factor 3D dome &amp; deterministic rays)</b>	
1	<code>function [drays] = Bvf3Dd(S)</code>
2	<code>cs = 20;npas=8;b=(0:360)*pi/180; % Parameter for the execution</code>
3	<code>Nc = size(S,2);Nt = S(Nc);R = S(2:Nc)-S(1:Nc-1);% Definition of the rings</code>
4	<code>r = sqrt(S/Nt);t = asin(r);h = cos(t);</code>
5	<code>for i= 1 : Nc; % Drawing the parallels in axnometric projection</code>
6	<code>x = r(i)*cos(b);y = r(i)*sin(b);z = ones(size(b,2))*h(i);</code>
7	<code>plot3(x,y,z,'k'); hold on;</code>
8	<code>end</code>
9	<code>la=zeros(Nt,1);lo=zeros(Nt,1);n=1;</code>
10	<code>for i = 1 : Nc-1; % Drawing the meridians segments</code>
11	<code>lp=0.;</code>
12	<code>for j = 1 : R(i);</code>
13	<code>longi = j*2*pi/R(i);</code>
14	<code>lai = t(i);las = t(i+1);pala = (las-lai)/npas;</code>
15	<code>mxi = sin(lai:pala:las)*cos(longi);</code>
16	<code>myi = sin(lai:pala:las)*sin(longi);</code>
17	<code>mzi = cos(lai:pala:las);</code>
18	<code>plot3(mxi',myi',mzi','k');hold on</code>
19	<code>if nargout &gt;0;</code>
20	<code>n=n+1;la(n)=(t(i+1)+t(i))/2;lo(n)=(longi+lp)/2;lp=longi;</code>
21	<code>end</code>
22	<code>end</code>
23	<code>end;</code>
24	<code>if nargout &gt; 0;drays=[la lo];end</code>
25	<code>title(['Equal view factor dome composed of ',num2str(Nt),' elements'],...</code>
26	<code>'fontsize',cs);Bwdo(50);hold on;axis equal;axis off;</code>
27	<code>end</code>

Table 12: Function **Bvf3Dd**. Dome mesh and eventually generation of deterministic rays

### 7.3. General procedures

The two following procedures are the same, except for the color of the dome. The blue is used for *EVF* situations and the red for *ESA* ones.

<b>Bbdo (blue opaque dome)</b>	
1	<code>function [xx,yy,zz] = Bwdo(n)</code>
2	<code>% Bwdo generate a white opaque dome (Beckers white dome)</code>
3	<code>% [X,Y,Z] = Bwdo(n) generates three (n+1)-by-(n+1)</code>
4	<code>% matrices so that SURF(X,Y,Z) produces a unit hemispherical dome.</code>
5	<code>%</code>
6	<code>% [X,Y,Z] = Bwdo uses n = 20.</code>
7	<code>%</code>
8	<code>% Bwdo(n) and just Bwdo graph the sphere as a SURFACE</code>
9	<code>% and do not return anything (nargout = 0).</code>
10	<code>%</code>
11	<code>if nargin == 0, n = 50; end % tests the presence of argument</code>
12	<code>theta = (-n:2:n)/n*pi;phi = (0:1:n)'/n*pi/2;</code>
13	<code>cosphi = cos(phi) ; cosphi(1) = 1; cosphi(n+1) = 0;</code>
14	<code>sintheta = sin(theta); sintheta(n+1) = 0;</code>
15	<code>scal = .99; % radius of the white dome</code>
16	<code>x = scal*cosphi*cos(theta);</code>
17	<code>y = scal*cosphi*sintheta;</code>
18	<code>z = scal*sin(phi)*ones(1,n+1);</code>
19	<code>blue=[0.7 0.95 0.95];colormap(blue);%blue dome or white: colormap(white)</code>
20	<code>if nargout == 0 % computed or returned result</code>
21	<code>surf(x,y,z, 'EdgeColor', 'none')</code>
22	<code>else</code>
23	<code>xx = x; yy = y; zz = z;</code>
24	<code>end</code>

Table 13: Function **Bbdo**: creates a blue opaque dome for hidden lines elimination (*EVF*)

<b>Bodo (red opaque dome)</b>	
1	<code>function [xx,yy,zz] = Bodo(n)</code>
2	<code>% Bwdo generate a white opaque dome (Beckers white dome)</code>
3	<code>% [X,Y,Z] = Bwdo(n) generates three (n+1)-by-(n+1)</code>
4	<code>% matrices so that SURF(X,Y,Z) produces a unit hemispherical dome.</code>
5	<code>%</code>
6	<code>% [X,Y,Z] = Bwdo uses n = 20.</code>
7	<code>%</code>
8	<code>% Bwdo(n) and just Bwdo graph the sphere as a SURFACE</code>
9	<code>% and do not return anything (nargout = 0).</code>
10	<code>%</code>
11	<code>if nargin == 0, n = 50; end % tests the presence of argument</code>
12	<code>theta = (-n:2:n)/n*pi;phi = (0:1:n)'/n*pi/2;</code>
13	<code>cosphi = cos(phi) ; cosphi(1) = 1; cosphi(n+1) = 0;</code>
14	<code>sintheta = sin(theta); sintheta(n+1) = 0;</code>
15	<code>scal = .985; % radius of the white dome</code>
16	<code>x = scal*cosphi*cos(theta);</code>
17	<code>y = scal*cosphi*sintheta;</code>
18	<code>z = scal*sin(phi)*ones(1,n+1);</code>
19	<code>ora=[1 0.8 0.7];colormap(ora); % orange dome</code>
20	<code>if nargout == 0 % computed or returned result</code>
21	<code>surf(x,y,z, 'EdgeColor', 'none')</code>
22	<code>else</code>
23	<code>xx = x; yy = y; zz = z;</code>
24	<code>end</code>

Table 14: Function **Bodo**: creates a red opaque dome for hidden lines elimination (*ESA*)

<b>Bwba (red opaque sphere)</b>	
1	<code>function [xx,yy,zz] = Bwba(n)</code>
2	<code>% Bwba generate a white opaque sphere (Beckers white ball)</code>
3	<code>% [X,Y,Z] = Bwba(n) generates three (n+1)-by-(n+1)</code>
4	<code>% matrices so that SURF(X,Y,Z) produces a unit sphere.</code>
5	<code>%</code>
6	<code>% [X,Y,Z] = Bwba uses n = §0.</code>
7	<code>%</code>
8	<code>% Bwba(n) and just Bwba graph the sphere as a SURFACE</code>
9	<code>% and do not return anything (nargout = 0).</code>
10	<code>%</code>
11	<code>if nargin == 0, n = 50; end % test of the presence of argument</code>
12	<code>theta = (-n:2:n)/n*pi;sintheta = sin(theta);</code>
13	<code>phi = (-n:2:n)'/n*pi/2;cosphi = cos(phi);</code>
14	<code>scal = .99; % radius of the white dome</code>
15	<code>x = scal*cosphi*cos(theta);</code>
16	<code>y = scal*cosphi*sintheta;</code>
17	<code>z = scal*sin(phi)*ones(1,n+1);</code>
18	<code>ora=[1 0.8 0.7];colormap(ora);</code>
19	<code>if nargout == 0 % computed or returned result</code>
20	<code>surf(x,y,z,'EdgeColor','none') % To see the sphere, replace none by k</code>
21	<code>else</code>
22	<code>xx = x; yy = y; zz = z;</code>
23	<code>end</code>

Table 15: Function **Bwba**. creates an red opaque sphere for hidden lines elimination (**ESA**)

The two following procedues are also the same except for the color of the dome supporting the rays. The blue is used for **EVF** domes (**Bsrays**) and the red for **ESA** spheres or domes (**Bvrays**).

<b>Bvrays (visualization of rays on spherical surface for ESA meshes)</b>	
1	<code>function [] = Bvrays(colo)</code>
2	<code>b=(0:5:360)*pi/180; % Definition of the base disk</code>
3	<code>x = cos(b);y = sin(b);z = zeros(size(b,2));plot3(x,y,z,'k'); hold on;</code>
4	<code>la = colo(:,1);lo=colo(:,2);</code>
5	<code>plot3(sin(la).*cos(lo),sin(la).*sin(lo),cos(la),'k'); hold on;</code>
6	<code>if la(size(la,1)) &gt; pi/2;</code>
7	<code>Bwba(50);hold on;axis equal;axis off; % White sphere</code>
8	<code>else</code>
9	<code>Bwdo(50);hold on;axis equal;axis off; % White dome</code>
10	<code>end</code>
11	<code>end</code>

Table 16: Function **Bvrays**. Representation of the spherical coordinates of the **ESA** rays

<b>Bsrays (visualization of rays on spherical surface for EVF meshes)</b>	
1	<code>function [] = Bvrays(colo)</code>
2	<code>b=(0:5:360)*pi/180; % Definition of the base disk</code>
3	<code>x = cos(b);y = sin(b);z = zeros(size(b,2));plot3(x,y,z,'k'); hold on;</code>
4	<code>la = colo(:,1);lo=colo(:,2);</code>
5	<code>plot3(sin(la).*cos(lo),sin(la).*sin(lo),cos(la),'k'); hold on;</code>
6	<code>if la(size(la,1)) &gt; pi/2;</code>
7	<code>Bwba(50);hold on;axis equal;axis off; % White sphere</code>
8	<code>else</code>
9	<code>Bwdo(50);hold on;axis equal;axis off; % White dome</code>
10	<code>end</code>
11	<code>end</code>

Table 17: Function **Bsrays**. Representation of the spherical coordinates of the **EVF** rays

## 7.4. Procedures

Name	Object	Input	Output
<b>Generation of cells sequence</b>			
<b>Bsamd</b>	<b>Equal solid angle (<i>ESA</i>)</b> dome mesh sequence	Number of cells default value: 145	Sequence of cells
<b>Bsams</b>	<i>ESA</i> sequence for a sphere	Number of cells default value: 290	Sequence of cells
<b>Bvfmd</b>	<b>Equal view factor</b> , equal aspect ratio ( <i>EVF</i> ) sequence	Number of cells, default value: 145	Sequence of cells
<b>Generation of rays and 3D representation of the mesh</b>			
<b>Bsa3D</b> <i>ESA</i>	Displays the dome and generates random rays	Sequence of cells	Image, matrix of colatitudes & longitudes
<b>Bsa</b> <i>ESA</i>	Generates random rays on the dome	Sequence of cells	matrix of spherical coordinates colatitudes & longitudes
<b>Bsa3Dd</b> <i>ESA</i>	Displays the dome and generates deterministic rays	Sequence of cells	Image, matrix of colatitudes & longitudes
<b>Bsa3Ds</b> <i>ESA</i>	Displays the sphere and generates random rays	Sequence of cells	Image, matrix of colatitudes & longitudes
<b>Bsas</b> <i>ESA</i>	Generates random rays on the sphere	Sequence of cells	Matrix of spherical coordinates colatitudes & longitudes
<b>Bsa3Dsd</b> <i>ESA</i>	Displays the sphere and generates deterministic rays	Sequence of cells	Image, matrix of colatitudes & longitudes
<b>Bvf3D</b> <i>EVF</i>	Displays the dome and generates random rays	Sequence of cells	Image, matrix of colatitudes & longitudes
<b>Bvf</b> <i>EVF</i>	Generates random rays	Sequence of cells	Matrix of spherical coordinates colatitudes & longitudes
<b>Bvf3Dd</b> <i>EVF</i>	Displays the dome and generates deterministic rays	Sequence of cells	Image, matrix of colatitudes & longitudes

Name	Object	Input	Output
<b>General procedures</b>			
<b>Bbdo</b> <i>EVF</i>	Creates a blue dome for hidden lines elimination	Size of the mesh Default value: 50	Blue dome drawing
<b>Bodo</b> <i>ESA</i>	Creates an red dome for hidden lines elimination	Size of the mesh Default value: 50	Red dome drawing
<b>Bwba</b> <i>ESA</i>	Creates a red sphere for hidden lines elimination	Size of the mesh Default value: 50	Red sphere drawing
<b>Bvrays</b> <i>ESA</i>	Draws previously computed rays	Matrix colatitudes & longitudes	Points on the dome or the sphere
<b>Bsrays</b> <i>EVF</i>	Draws previously computed rays	Matrix colatitudes & longitudes	Points on the dome for <i>EVF</i> visualizations



## 8. References

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