

# Impact of solar energy on cities sustainability

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**ABSTRACT:** *At the building scale, solar energy is concerned with passive and active design and with daylight supply. The management of these resources is relatively well handled by the principal thermal regulations, but only for separate buildings.*

*At the urban scale, multiple interactions make very difficult to maintain the same kind of analysis. Other methods must be developed to take into account the complex dynamic behaviour of very different buildings facing each others, forming streets and places, sharing walls and heating systems. Several intermediate objectives are emerging both from the scientific research and from the regulation progress, as solar impact studies, quantification of the solar potential of urban areas, contribution of the radiative transfers to the urban climate.*

*The three preliminary questions we propose to address here are the following:*

- *What is the correct scale for such an analysis?*
- *How could a satisfactory 3D model be defined?*
- *How could a properly simplified radiosity simulation be performed?*

*As cities are probably the source of the most important energy saving potential for the near future, these questions are fundamental for the urban systems engineers concerned with global energetic efficiency and for the architects involved in sustainable urban planning and refurbishment.*

*Keywords: solar energy, urban scale*

## 1. INTRODUCTION

Among the various issues concerned by urban physics, the interaction between the city and the atmosphere is probably the most difficult. It involves highly coupled phenomena: radiative exchanges, aerodynamics and water cycle. Since the 1970s, the pioneering work of environmental physicists such as JL Monteith [1], GS Campbell [2] and TR Oke [3] helped establishing energy balances from the scale of the entire planet until the territorial and urban microclimates ones. In particular, TR Oke has long explored the phenomenon of urban heat island (UHI); this research has been developed until today, gradually incorporating the technical developments of the resources of measurement and simulation. However, this global approach does not take into account the geometry of the studied scene. A second approach, which gave most of its results in the 90's, consisted of studying very simple geometric patterns, often narrow streets (canyon effect), in order to link basic geometric parameters (street width, building heights, orientation) with all physical phenomena that produce the surfaces and air temperatures. This local approach was used to compare simulations with measurements, but the complexity of the couplings does not allow considering larger neighborhoods.

In recent years some very significant progresses were: the maturing of the simulations for realistic rendering, the creation of 3D models of urban ensembles with different levels of detail, the contribution of second generation satellites for the measurements. We propose to describe the full radiative problem, but with minimal couplings, so as to make possible the simulation of complex geometric designs and to obtain initial urban quantifications.

## 2. PHYSICAL CONSIDERATIONS

The sun, whose surface can be treated as a blackbody of about 6000 K, emits energy that at the Earth's surface is divided almost equally between visible and near infrared ( $< 3 \mu$ ). Part of these rays are scattered by the sky, which can be considered as a secondary source of illumination, with similar spectral characteristics. The components of direct and diffuse irradiance on clear days can be calculated fairly accurately from Beer's law and the measurements of Liu & Jordan (taking into account the atmospheric absorption, which varies principally with the height of the sun, the sky and the altitude) [2]. Necessary parameters are the latitude and time of day (solar path), altitude, orientation and inclination of the surface under investigation. In urban applications, it is important to keep separate the direct and diffuse components, which do not interact the same way with the geometry. For a given surface, the sun is visible or hidden depending on its position, and therefore the time and day (visibility function). The sky is more or less visible (sky view factor), regardless of time, latitude and even orientation. From the perspective of an energy balance over the long term (monthly, seasonal or annual), cloud cover can then be taken into account statistically, as a correction factor acting differently on the two components.

The surfaces of the urban scene (soil, facades and roofs), warmed by the sun and possible internal inputs (heating in winter), at most reach temperatures of several tens of centigrade degrees. They therefore emit far Infrared (between 3 and 100  $\mu$ ) with an emissivity equal to 0.95 almost always: with the exception of large metal surfaces, seldom seen in the urban environment, they can be considered blackbodies which radiate according to Stefan-Boltzmann law ( $B = \epsilon \sigma T^4$ ). The atmosphere

plays a more complex role. On clear days, the temperature of the lower layers, the most effective, is on average some 20° lower than the ground one, but with a large angular variation (the direction of the zenith is much colder than the horizon one) [1]. Between about 8 and 12  $\mu$ , the atmosphere has a spectral window, which lets escape most of the radiation to space. In the presence of clouds, this window closes, which amplifies the greenhouse effect. Taking into account the atmosphere in a balance of energy in the long term is the most critical issue we have to deal with.

Despite all these difficulties, a considerable advantage is the separation between the emission spectra of sun and sky, on the one hand, and heated surfaces on the other one (including the atmosphere, modeled as a canopy): 95% of the radiated energy is, respectively, below 3  $\mu$  for the former (we speak now of short waves), and beyond for the last (now classified as long waves). So, we are facing two well differentiated issues. For short waves, the emission is limited to sun and sky, and the materials are characterized by their reflection coefficient (for thermal issues, the color can be expressed as different gray levels averaged over the short-wave) and, for translucent materials, by a transmission coefficient. The remaining energy is absorbed by the material and contributes to its heating. In the long waves, all surfaces emit with an emissivity close to unity. According to Kirchhoff's law, they also absorb almost everything they receive, and reflection can be neglected. Surface temperatures are the link between the two phenomena; they are therefore the main parameters of the problem.

### 3. OBJECTIVES OF THE STUDY

To assess the necessary CAD model, we must first clarify the five principal issues surrounding solar energy at the urban scale.

- Urban comfort. At street level, you need shade in summer (especially in southern cities) and sunny places in winter (especially in northern cities). This problem is the simplest one, because it only requires considering the masks that produce buildings seen from urban land. It is integrated with many urban regulations, in the form of limitation on the height of buildings, often depending on the width of the streets.

- Solar potential. It concerns the thermal and photovoltaic solar panels on roofs and possibly on top of well-oriented façades. The current application requires both a comprehensive quantification of this potential, at least at the neighborhood level, but also detailed knowledge of the best locations on an architectural scale, for individual or shared use. In the case of photovoltaic, the high sensitivity to shading, even partial and temporary [4], requires a model taking into account all the details of the roof (e.g., elevator shafts and chimneys).

- Daylight supply. It concerns the direct and diffuse components of visible light reaching the windows. Without prejudging the use of the light inside buildings, we can consider that a good illumination on the windows saves artificial lighting

and provides better building comfort. Reflection on nearby buildings and ground plays an important role.

- Solar thermal contributions. They affect mainly the façades, distinguishing windowing and opaque walls. They allow specifying the energy balance of buildings in urban areas. To do this, surface temperatures and external air temperature must be specified. Taking into account the cloud cover is essential for annual or seasonal balances.

- Urban climate. This is the most difficult point since wind and water cycle couplings must be taken into account. At first glance, it would already be useful to evaluate only the participation of radiative exchanges in urban climate.

### 4. THE CAD MODEL

The CAD model of an urban area should contain the façades with windows, the roofs and the ground with its relief. It must be greatly simplified, but so that the great surfaces are correctly oriented and seamless. These requirements do not correspond to the hierarchy proposed by the *Levels Of Detail* [5], mainly defined for visual quality. It may be recalled that the LOD1 corresponds to the extruded cadastre (prismatic buildings with flat roofs), the LOD2 offers many types of roofs, but the windowing appear only as textures applied to façades and the LOD3 is already much too detailed. Currently there are no realized 3D city models for physical simulation. The first objective of this paper is to show that such a model is necessary and feasible. In addition, the objective of radiation exchange is a good starting point: it is both the most demanding on the geometry, and least sensitive for the extension. We therefore hope that the work presented here serves as a prototype for a CAD model able to be generalized to all urban physical studies.

An original 3D model was built, which represents the center of the city of Compiègne [6]. In its current state (figure 1), it consists of just over 20,000 triangles in the STL standard format. We tested it with the simulation of direct solar radiation on clear days, and with the sky view factor. These two parameters have the advantage of being simple to calculate (we use the software Heliodon 2 [7]) and representing links between radiative exchanges and geometry.

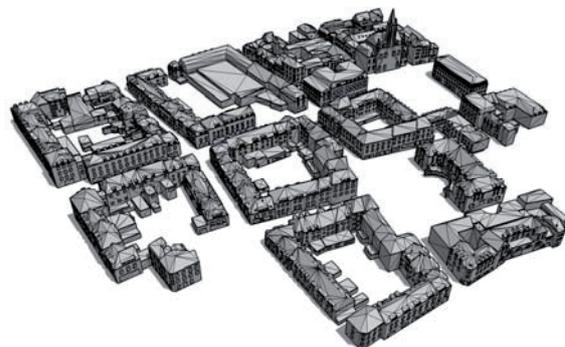


Figure 1: The present CAD model of Compiègne centre, with tilted roofs, façades and windows (20 000 triangles)

As a first result, we observe that the calculation of solar annual potential (omitting the overcast) hardly changes when the roofs are reduced to flat planes (level LOD1). This constancy, verified in several configurations, is true only for an annual balance. In fact, small gains in summer are offset by significant losses in winter. Moreover, the level LOD1 does not allow locating the best slopes for photovoltaic systems, and thus can not be accepted. In contrast, a model much simpler than the present, only taking into account the average inclination street by street, without worrying about connection surfaces, seems amply sufficient, and allows to substantially reduce the size of our model.

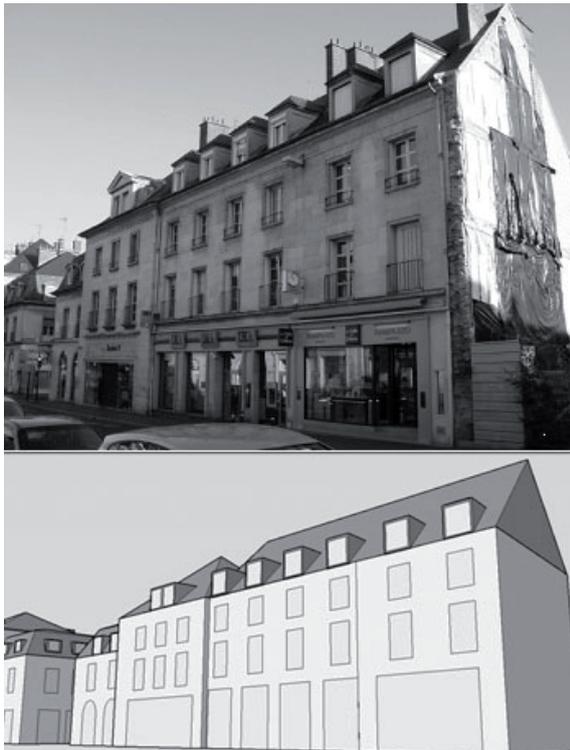


Figure 2: Level of detail of the CAD model; façades and windows are simple superposed rectangles

The windows are plane rectangles placed a centimeter outside the walls, which avoids to drill them (figure 2). This simplification reduces consequently the size of the model. Moreover, the meshes are realized much more quickly. However, the thickness of the walls is being neglected, whereas it significantly reduces the view of the sky and direct sunlight inside, especially in old buildings as is the case in our model. We have therefore established a *posteriori* correction tables, depending on the thickness of the wall, shape and orientation of the window, the latitude considered. With these initial studies, we gradually simplify the CAD model, while retaining the best quality simulations. We can already conclude that this model of urban expansion is affordable with tens of thousands of triangles (STL), separated in different layers (figure3), and a semantics that associates to each surface geometric and physical characteristics

(thickness, color, emissivity, heat capacity and conductance, ...).

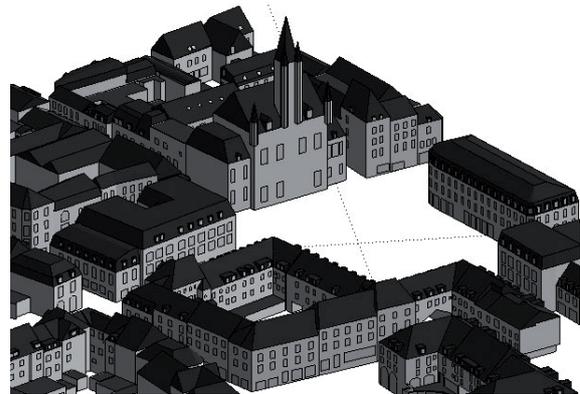


Figure 3: The CAD model, with different layers for roofs, façades and windows

## 5. BOUNDARY OF THE 3D MODEL

The CAD model for solar studies has the advantage of being insensitive to scale. As long as its boundary is well defined, the simulation results are not affected by a subsequent enlargement of the study field. To do this, the objects outside the modeled area should be distinguished. Nearby buildings, for which the masking effect varies greatly when we move in the study area, must be modeled in a simple manner, with flat roofs with a height corresponding to actual buildings tops. This forms the boundary of the model. The large and distant buildings, as the terrain and mountains, have the characteristic of maintaining their appearance when we are traveling in the modeled area. In order to take them into account in the simplest way, we define with them a second boundary, the skyline, which surrounds the first one in the distance.

With these rules, we can choose freely the extension of the modeled area. We have decided to consider three scales that correspond to different activities and, often, to different regulations. The scale of the building (micro) is that of architects. It may eventually extend to the urban block, separated from other buildings by streets. The scale of the city (meso) extends from the area comprising at least a few blocks and their boundaries until the entire city with its adjoining suburbs. The territory scale (macro) is that of meteorologists and geographers. For us, it corresponds essentially to a given climate, the one of the studied city.

We are interested here in the meso scale, and in the data to be exchanged with the two other scales to better understand the radiative exchanges at the urban scale and the usefulness of their simulation.

The micro scale needs to know the masks, the availability of sunlight and skylight allowed by the urban context, but also the particular climate, and local variations in air temperature. If practices such as exterior insulation, air conditioning or solar energy capture are becoming usual, it is necessary to study their impact on the urban radiative exchanges.

The macro scale provides climate data, but it also receives the influence of the city. Satellites observe higher temperatures in the urban centers, which are the hallmark of the heat island. However, remote measurements of the radiation are necessarily indirect, and evaluation of the city albedo is a demand of this scale to the meso scale.

So, the meso scale is the heart of the so defined multiscale problem. For software able to evaluate the radiative exchanges there will be three main tasks:

- Sharing information (air and surfaces temperatures) with the two other scales, to allow greater precision of the simulations by feedback;
- Conducting impact studies, properly urban, allowing to choose among solutions for urban development, those who have the best consequences for the climate and the comfort of the inhabitants (according to the five objectives listed above);
- Participating in the development of relevant urban regulations, as a generalization of previous impact studies.

## 6. SHORT WAVES SIMULATION MODEL

This kind of simulation takes benefit of the methods developed in the frame of picture rendering and virtual reality. Therefore, it is possible today to develop efficient software for applications proceeding from engineering disciplines as different as astronautics and architecture.

Because they are submitted to severe thermal loads, the space structures need a careful evaluation of their thermal response in order to ensure the integrity of the spacecraft components. It is now accepted that the finite elements models allow obtaining this required precision. But the computation is very heavy. The challenge is then to improve the performances of the algorithms and to better control the precision of the results. This error control can be achieved by combining light and importance transport formulations [8].

When dealing with urban areas, even if the basic problem is the same, new difficulties appear: the geometrical model involves much more components and it is not easy to define the boundary conditions. As a consequence, it is difficult to obtain a global solution with a high precision everywhere. As for the space application, combining estimates of light and importance to drive the global solution help to achieve the required precision in the right location.

The relevant quantity used to describe radiant energy transfer is radiance. Radiance is defined as the amount of energy traveling at some point, in a specified direction, per unit time, per unit area perpendicular to the direction of travel, and per solid angle [9]. In radiometry, it is measured in W/m<sup>2</sup>/sr (sr means steradian; it is used to measure solid angles).

This quantity is a function of the position and the ray's orientation, but it also depends on wavelength and time (seven variables). In papers dealing with vision, the terminology used to describe this quantity is the plenoptic function [10].

Because of its complex nature, it is difficult to solve the general radiative transfer equation.

However, a simplified discrete formulation, widely known as radiosity equation, first invented by thermal engineers in the 1950s [11] was later proposed to solve the global illumination problems [12].

$$\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix} + \begin{pmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1N} \\ \rho_2 F_{21} & \rho_2 F_{22} & & \vdots \\ \vdots & & & \vdots \\ \rho_N F_{N1} & \cdots & \cdots & \rho_N F_{NN} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix}$$

$$MB = E \quad ; \quad M_{ij} = \delta_{ij} - \rho_i F_{ij}$$

$B_i$  is the radiosity or the radiant flux per unit area on patch  $i$  (expressed in Wm<sup>2</sup>),  $E_i$  the exitance (or emissive power, or radiant exitance),  $\rho_i$  the hemispherical diffuse reflectance and  $F_{ij}$  the form factor between patch  $i$  and patch  $j$ . Radiosity is the radiometric quantity that is best suited for quantifying the illumination in a diffuse scene.

This discrete formulation is derived from the global illumination equation by making the following assumptions:

- the environment is a collection of a finite number  $N$  of small diffusively reflecting patches with uniform temperature and radiosity;
- the solution is carried on in an enclosure; i.e. the hemispherical directions around any point in the environment are assumed to be covered by one or more of the patches and every patch may be assumed to occupy a solid angle in the hemisphere over a surface point.

The radiosity formulation leads to a linear system of equations for which many algorithms are available. Several iterative methods can be used; see for instance, the Southwell relaxation method [9] or the Gauss-Seidel and Jacobi iterative methods [13]. However, the heaviest part of this computation lies in the form factors evaluation, both with respect to CPU time and data storage. In practice, the iterative solutions require computing only one line of the matrix at each iteration.

The drawback of the global algorithms is that they compute the radiosity of the masked surfaces with a useless accuracy while the visible surfaces would maybe require an additional computational effort. In order to overcome this difficulty, a solution has been found, based on previous work on the simulation of the transport of neutrons in the nuclear reactors. The idea is to identify the interactions which need to be accurately computed and to neglect the secondary interactions. If we consider the case of a street in the urban model, the source of light is either the sky or the sun. If we are interested in a given street, it is useless to accurately compute the lighting of the other streets. In this case, a particular region of the space is more important than the others. A global algorithm is clearly inappropriate. This example illustrates the notions of importance.

Therefore, another way to make radiosity practical for complex environments is to incorporate a notion of view dependence. In illumination theory, the importance or potential of a surface patch is a

measure of the impact on the final picture of a unit radiosity on that surface. Its equation can be formed by expressing the total importance of a patch as the sum of an intrinsic importance  $W_i$  and that contributed by the other patches.

$$Y_i = W_i + \sum_{j=1}^N \rho_j F_{ji} Y_j$$

$$M^T Y = W \quad ; \quad M_{ji} = \delta_{ji} - \rho_j F_{ji}$$

This relation is very similar to the radiosity equation and the way to derive it is quite the same [8], [14], [15]. To explain it, we can write the following relations by using well known results from linear algebra. Each scalar product  $W^T B$  related to the solution of a linear system  $MB = E$  can be obtained as a scalar product of the source term  $E$  related to the solution of the adjoint system of linear equation  $M^T Y = W$  with source term  $W$ :

$$W^T B = (M^T Y)^T B = Y^T MB = Y^T E$$

We can interpret that as follows. Consider the power  $P_k$  emitted by the patch  $k$ .  $P_k$  can be written as a scalar product ( $A_k$  denotes the area of patch  $k$ ):

$$P_k = A_k B_k = W^T B \quad \text{with} \quad W_i = A_i \delta_{ik} \partial$$

All components of the direct importance vector  $W$  are 0, except the  $k^{\text{th}}$  component, which is equal to  $W_k = A_k$ .  $P_k$  can also be obtained as:

$$P_k = Y^T E = \sum_i Y_i E_i$$

It is a weighted sum of the self-emitted radiosities at the light sources in the scene. The solution of the adjoint system indicates to what extent each light source contributes to the radiosity at  $k$ . The importance  $Y$  is also called potential [13].

The illumination can be obtained by solving either one of the two adjoint equations. The previous example also explains how to define the intrinsic importance terms  $W$ . The real benefit of this dual formulation is that, if approximate solutions are available for both equations, they can be combined into a new solution with higher accuracy than either component alone. We can also use the concept of importance and the adjoint system to establish a measure of the error involved in the computation.

## 7. LONG WAVES SIMULATION MODEL

The radiosity method is valid for instantaneous exchanges of light, when most elements of the scene are pure diffuse reflectors. Here, the short waves radiative sources are the sun and the sky. The simulations can provide the values of radiosities on all the surfaces. The incident energy is stored in the material and brought back to the exterior as long wave radiation. In this case, most elements behave

as blackbody's emitters. This new problem cannot be considered as a steady radiative exchange; it needs to introduce additional terms, as explained hereafter. All the variables of the model are time dependent.

Fortunately, as the short waves problem is independent of surfaces temperatures, it can be solved first, as explained in the previous section, and the resulting irradiances are then considered as simple thermal loads in the long waves balance explained in this section.

In solids, the solution of the thermal problem is derived from the principle of conservation of energy over the volume.

$$\text{div}[k \text{ grad } T] + Q = \gamma c \frac{\partial T}{\partial t}$$

The specific rate of heat generation in the volume is defined by  $Q$  ( $\text{W m}^{-3}$ );  $T$  is the temperature measured in Kelvin degrees (K),  $t$  the time in seconds (s),  $k$  the thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ ),  $c$  the specific heat ( $\text{J kg}^{-1} \text{K}^{-1}$ ) and  $\gamma$  the density ( $\text{kg/m}^3$ ).

To develop a numerical solution, like in Esarad (ESA's standard thermal radiation analysis software, supporting both the Monte Carlo ray-tracing and matrix methods) - Esatan (ESA's standard tool for thermal analysis, « the » European tool) [16] [17], the model can be decomposed in isothermal elements called thermal nodes. It implies that the properties are constant across a thermal node. In the lumped thermal parameters method, a nonlinear system of differential equations is assembled and solved by establishing the heat balance at each isothermal node (Fourier's law and Stefan-Boltzmann law). For node  $i$ :

$$\sum_{j=1}^N K_{ji} (T_j - T_i) + \sum_{j=1}^N R_{ji} (T_j^4 - T_i^4) + \sum_{j=1}^N H_{ji} (T_j - T_i) + Q_i = m_i c_i \frac{\partial T_i}{\partial t}$$

$K_{ji}$  represents the conductive links ( $\text{WK}^{-1}$ ),  $R_{ji}$  are the radiative links ( $\text{WK}^{-4}$ ),  $H_{ji}$  is the convective heat transfer to fluid reference nodes ( $\text{WK}^{-1}$ ),  $Q_i$  is the power dissipation or heat source at the node (W),  $m_i$  is the node's lumped mass (kg) and  $c_i$  the node's specific heat ( $\text{Jkg}^{-1}\text{K}^{-1}$ ). The product  $c_i m_i = C_i$  is the thermal capacitance ( $\text{JK}^{-1}$ ). The values  $C_i$ ,  $K_{ji}$ ,  $H_{ji}$  and  $R_{ji}$  depend on the physical properties and the geometry of the system and can be variable with the time. The radiative term is deduced from the matrix of radiosity and, in general, the numerical algorithms used to solve the network must linearize the radiative term.

The integration of the system provides the temperature vector at different times. Two standard methods able to complete this task are the explicit forward differences method and the Crank-Nicholson method.

## 8. CONCLUSIONS

In space structure, the preference is generally given to global radiosity algorithms because they are view-independent and compute the radiosity with a constant, uniform accuracy throughout the model. The quality of the solution only depends on the capability of the *a priori* defined mesh to meet the characteristics of the specific problem to be solved. In most situations, however, certain regions of the model are more sensitive to small temperature variations. This is the reason why a precise control of the error is needed.

The view-dependent algorithms can compute the radiosity of the surfaces which are visible by the receiver (eye of the observer, camera, and radiometer). The surfaces that are not visible from the receiver and have no influence on the visible surfaces are neglected. When the analysis is focused on a specific building or a small area inside the city, view-dependent solutions are obviously more efficient.

The discrete dynamic thermal equations are not easy to solve because of the high number of variables. However, the coupling is weak in the urban model. Conduction is generally limited to single buildings and radiation to street neighboring constructions. In the present model, the convective idealization is very coarse, but it is the only link with the climatic data and the upper level model.

The key point for the optimization of the computation is the handling of the geometric data. In the case of the urban modeling, it is easy to take into account the topology of the city which refers to 2D maps.

According to the present state of the art in heat transfer problems, it seems realistic to consider simulations involving tens of thousands of patches. Decoupled resolution for short waves and then for long waves should allow proper assessment of the radiative exchanges on a CAD model such as the one of Compiègne here presented.

With respect to global and local methods recalled in the introduction, the big advantage is to actually handle the urban scale (meso), making possible to exchange information with the micro and macro scales. This should allow both improving our knowledge of urban physics, but also advising better the architects in their urban projects, in order to better quantify their contribution to the sustainable city in terms of its interaction with atmosphere and climate.

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