URBAN OUTLINES 2D ABSTRACTION FOR FLEXIBLE AND COMPREHENSIVE ANALYSIS OF THERMAL EXCHANGES

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ABSTRACT

Thermal transfer modelling in the city involves several aspects: short waves radiative heat transfers related to the solar beam, short waves and long waves radiative transfers between the urban area and the sky, diffuse exchanges between buildings, conductive phenomena in the constructions, convective exchanges with ambient atmosphere and capacity of the objects to store energy. Strong difficulties have to be overcome to achieve this kind of simulation: complexity and dimension of the geometrical model, specification of initial and boundary conditions and heaviness of non lineal unsteady computations. The management of this problem needs discretizations error evaluation for the control of the solution.

In the development of the rendering techniques, some aspects have already been successfully addressed for a long time. In the radiosity problem, for example, the use of importance – the quantity dual to radiosity obtained by the solution of the adjoint problem – allows not only to speed up the solution, but also to introduce a control of its quality.

The radiosity method is valid for instantaneous exchanges of light, when most elements of the scene are pure diffuse reflectors. The incident energy is stored in the material and brought back to the exterior as long wave radiation. In this case, the elements behave as blackbody's emitters. This second problem cannot be considered as a steady radiative exchange; it needs to introduce additional terms that make the variables of the model time dependent.

Fortunately, as the short waves problem is independent of surfaces temperatures, it can be solved first, and the resulting irradiances are then considered as simple thermal loads in the long waves balance.

Assuming that convenient software is available, we still need to train the potential users to ensure the correctness of their results with an acceptable cost. It is then necessary to give them good skills to interpret the different situations they will meet. In the University, it is also convenient to educate the students in the management of the different techniques or algorithms.

We propose a simplified 2D program where most situations of thermal exchanges in the city can be easily reproduced and clearly exhibited. This software allows evaluating a wide variety of situations in an interactive way and with easy modifications of both the geometrical and physical data.

INTRODUCTION

This paper is devoted to the presentation of the radiosity method in 2D. This kind of presentation was already performed several years ago in in order to make the radiosity concept easier to understand [1]. The second idea is to propose to the students a software tool to help them to understand and experiment the methods and algorithms of global illumination and thermal exchanges.

FORM FACTORS

The view factor, also called form factor, is a pure geometric quantity even if its definition is based on energetic theory. It specifies the fraction of energy leaving a surface Q_i that reaches another surface Q_j [2].

$$F_{ij} = \frac{1}{A_i} \int_{x \in Q_i} \int_{y \in Q_j} \frac{\cos\theta_x \cos\theta_y}{\pi r^2} V_{ij}(x,y)\, dydx \tag{1}$$

The factor F_{ij} connects two patches: Q_i with area A_i and Q_j with area A_j (figure 1). The angle between the ray r and the normal to patch Q_i is denoted θ_x while the angle between the ray and the normal to patch Q_j is denoted θ_y. The symmetry of the integral yields to the property of reciprocity discovered by Lambert in 1760 [3]:

$$A_i F_{ij} = A_j F_{ji} \tag{2}$$

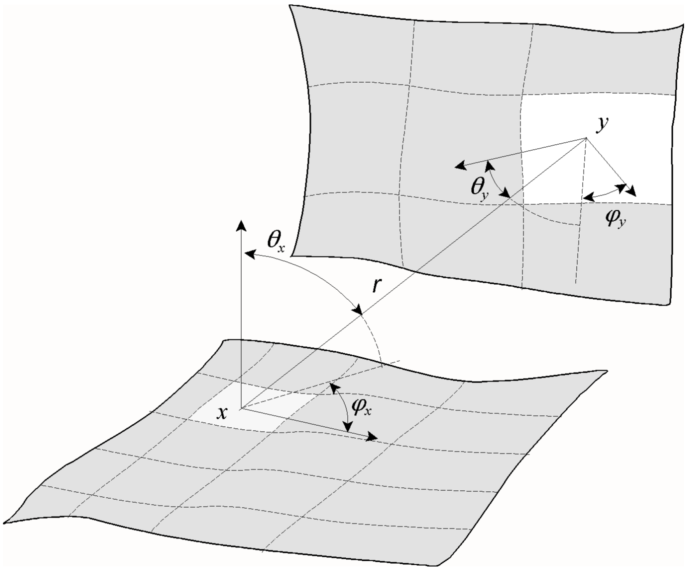

Figure 1: Definition of the form factor, after [2, 4]

The coefficient $V_{ij}(x, y)$ is the visibility function between the points x and y located on Q_i and Q_j; it takes only the values 1 (visible) or 0 (occluded). Explicit solutions of the view factor exist for a few numbers of particular configurations [2, 5] but computing the differential form represented by the inner integral is rather simple, at least if it is not necessary to solve a visibility problem.

The inner integral represents the point to area form factor. In 2D it is given by a formula:

$$F_{dL-j} = \int_{y \in L_j} \frac{\cos\theta_{dL} \cos\theta_j}{2r} dy \tag{3}$$

According to figure 2, it is simplified in:

$$F_{dL-j} = \frac{1}{2}\left(\frac{x_1}{r_1} - \frac{x_0}{r_0}\right) \tag{4}$$

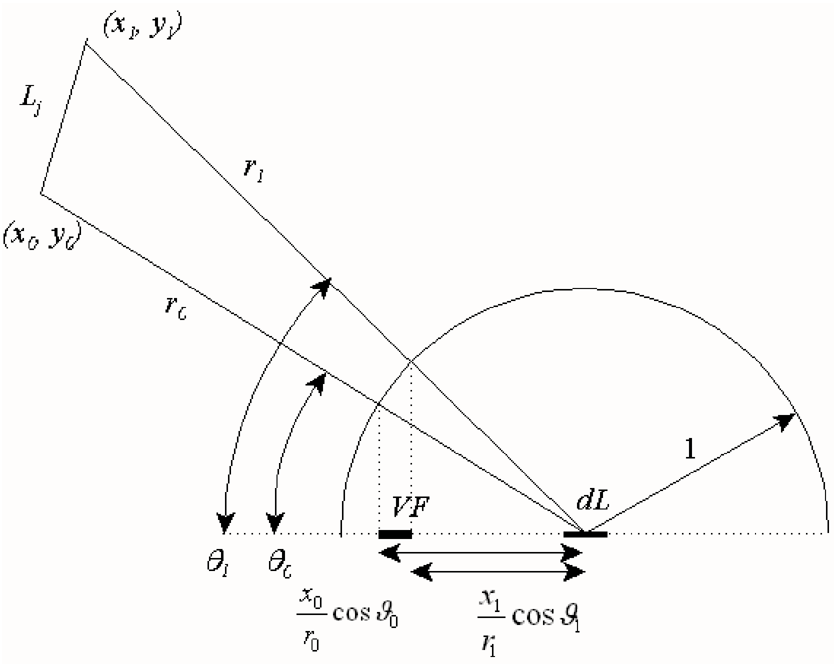

Figure 2: Computation of the point to area form factors

RADIOSITY EQUATION

The discrete formulation of the global illumination problem is given by the radiosity equations:

$$\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix} + \begin{pmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1N} \\ \rho_2 F_{21} & \rho_2 F_{22} & & \vdots \\ \vdots & & & \vdots \\ \rho_N F_{N1} & \cdots & \cdots & \rho_N F_{NN} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} \tag{5}$$

The B_i ,E_i and ρ_i terms represent respectively, the radiosity, the exitance and the reflectance of the element *i*,. The radiosity equation can be written as a system of *N* linear equations easy to solve with standard methods because the matrix *M* is well conditioned.

$$MB = E \quad ; \quad M_{ij} = \delta_{ij} - \rho_i F_{ij} \tag{6}$$

GEOMETRIC MODEL

The geometry is described by simple straight lines segments. These entities are supporting the finite element mesh. The mesh can be uniform or variable. Radiosity, exitance and reflectance are assumed to be constant on each finite element. Instead of computing the true form factor, the point to area form factor is computed in the Gauss points of the elements and then integrated. It can be shown that in most situations a single Gauss point per element is sufficient. The number *N* of finite elements is determining the size of the system of equations. Different geometric models are shown in figures 3, 5 and 6.

CLOSURE AND RECIPROCITY

To be consistent, the radiosity equations have to fulfill two important properties. The closure property specifies that the sum of the form factors relative to a single patch must be equal to 1 in a closed space. This condition is easily fulfilled in 2D. The reciprocity condition presented in equation (2), is satisfied only if the number of Gauss points is sufficient. In the particular situation where all the elements have the same size, the radiosity matrix is symmetric. It is shown below for a problem of 12 variables where the lack of symmetry indicates the approximation of the reciprocity condition.

0	0	0	0.0358	0.0738	0.0703	0.0965	0.1414	0.1644	0.0391	0.1023	0.2764
0	0	0	0.0840	0.1160	0.0764	0.1414	0.1644	0.1414	0.0764	0.1160	0.0840
0	0	0	0.2764	0.1023	0.0391	0.1644	0.1414	0.0965	0.0703	0.0738	0.0358
0.0391	0.1023	0.2764	0	0	0	0.0358	0.0738	0.0703	0.0965	0.1414	0.1644
0.0764	0.1160	0.0840	0	0	0	0.0840	0.1160	0.0764	0.1414	0.1644	0.1414
0.0703	0.0738	0.0358	0	0	0	0.2764	0.1023	0.0391	0.1644	0.1414	0.0965
0.0965	0.1414	0.1644	0.0391	0.1023	0.2764	0	0	0	0.0358	0.0738	0.0703
0.1414	0.1644	0.1414	0.0764	0.1160	0.0840	0	0	0	0.0840	0.1160	0.0764
0.1644	0.1414	0.0965	0.0703	0.0738	0.0358	0	0	0	0.2764	0.1023	0.0391
0.0358	0.0738	0.0703	0.0965	0.1414	0.1644	0.0391	0.1023	0.2764	0	0	0
0.0840	0.1160	0.0764	0.1414	0.1644	0.1414	0.0764	0.1160	0.0840	0	0	0
0.2764	0.1023	0.0391	0.1644	0.1414	0.0965	0.0703	0.0738	0.0358	0	0	0

Table 1: Form factors matrix for the mesh of figure 3 involving 12 elements

CONVERGENCE

We consider the different n orders of reflections. For the first one, we obtain ρ *E*, with E, the total exitance. For the second one, we obtain $\rho^2 E$, and, then, $\rho^3 E$ …

When the equilibrium is achieved, the total power *P* emitted by all the walls and boundaries is equal to the sum of the initial exitance and the infinite number of reflections

$$P = \left(1 + \rho + \rho^2 + \rho^3 + \ldots\right) E \tag{7}$$

The limit of the previous series leads to a simple formula

$$\forall \rho \in [0,1] \quad , \quad \sum_{n=0}^{\infty} \rho^n = \frac{1}{1-\rho} \quad \rightarrow \quad P = \frac{E}{1-\rho} \tag{8}$$

This relation is valid if the domain is closed and if all the reflection coefficients are the same.

APPLICATIONS

In all the examples, we will consider a square box (3m x 3m) as shown in figure 3. The same figure shows the boundary conditions. The exitance is defined on the centre of the top (roof) side and set equal to 1 Wm^{-1}. In all the tests, the finite element mesh is uniform. The reflexion coefficients are all equal to 0.5. The radiosity is assumed to be constant. The results are displayed showing the evolution of the radiosity along the boundary, from the bottom left vertex. The function abscise varies from zero to the perimeter of the domain (12 m).

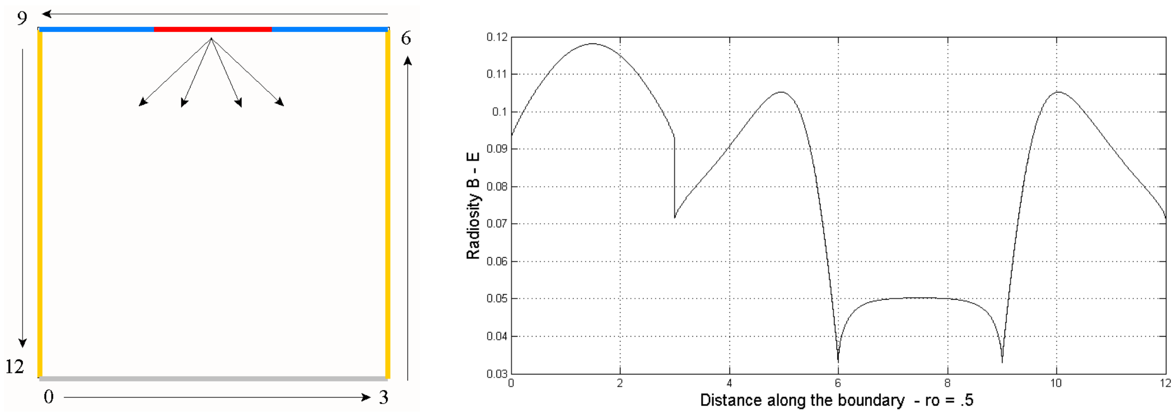

Figure 3: Mesh, boundary conditions, radiosity: 2400 elements, P = 2 W, CPU = 84 (59+25) seconds

A coarse mesh is built with 12 elements (3 per side). The coarse mesh results displayed in dashed line (figure 4) are compared to the exact solution. The imprecision of the result is reflected by the error on the total power.

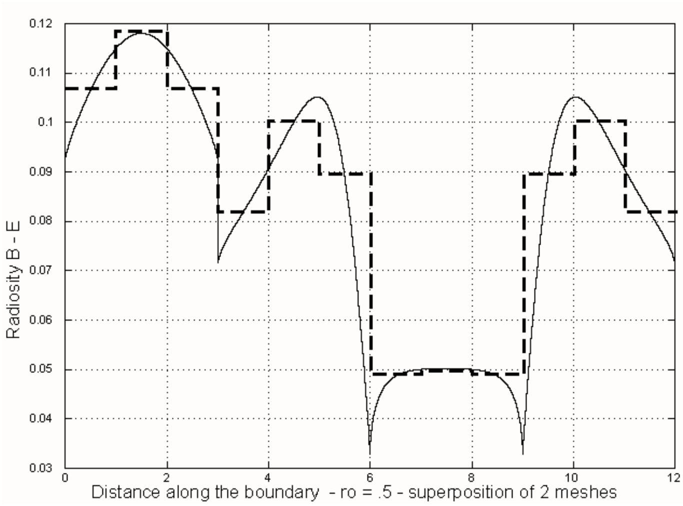

Figure 4: Radiosity for the coarse mesh, 12 elements (dashed line) versus exact solution, P = 2.0328 W.

This kind of simulation helps to understand the distribution of radiosity along the boundary of the domain. We observe that there is no difficulty to take into account the discontinuities of the solution. In the two following examples, the total exitance is equal to 1. The power is then converging to the same value of 2 *W*.

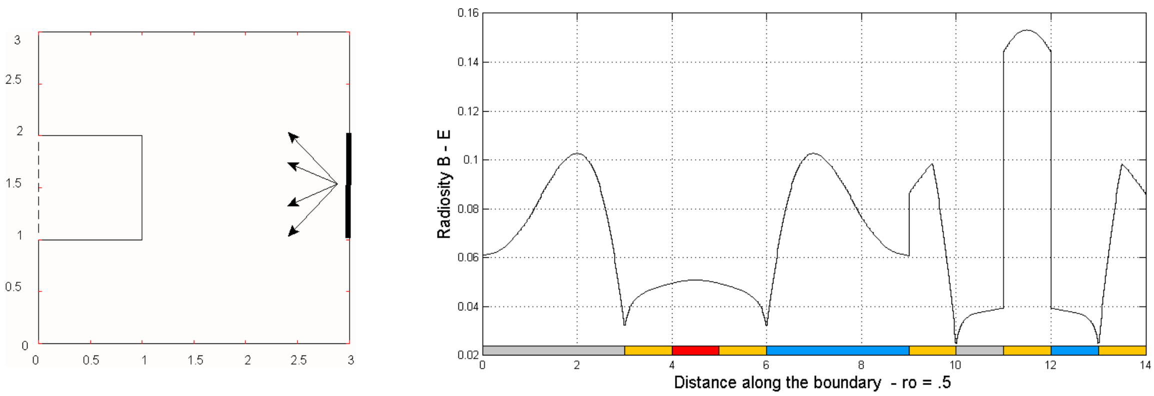

Figure 5 : 840 elements, P=2.000 W, CPU 8 seconds

We also observe that the CPU time is very acceptable. This kind of simulation helps to understand the distribution of radiosity along the boundary of the domain. We observe that there is no difficulty to take into account the discontinuities of the solution.

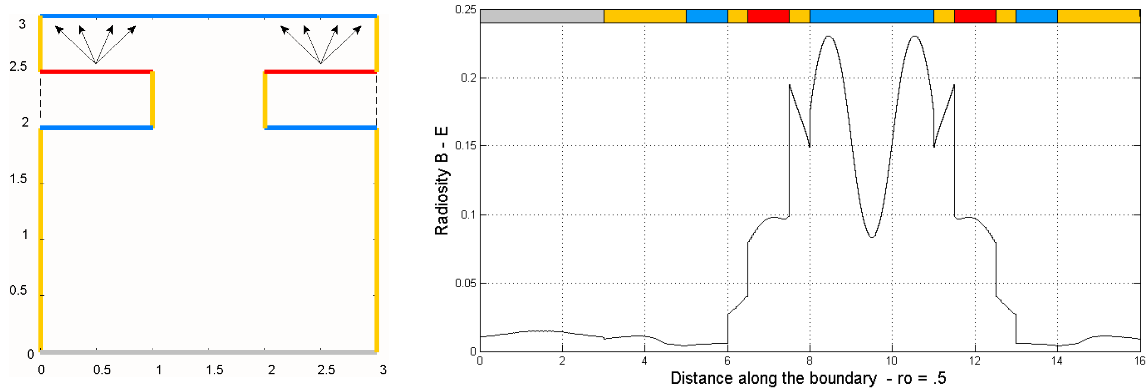

Figure 6: 800 elements, P=1.9997 W, CPU 8 seconds

CONCLUSION

The software developed to perform 2D radiosity analyses helps to understand the properties of the methods used to compute global illumination and the behavior of the algorithms used to obtain the solution. The cost of the solution is very low and allows then to compare many results. The convergence test performed on the total power in a close domain is very reliable and easy to use. Consequently, complicated geometric configurations can be handled. An important advantage of the 2D study is that we can represent any solution on graphics easy to interpret, for teaching purposes, but also to better design complex algorithms that will subsequently be implemented in 3D.

PERSPECTIVES

Due to the easiness to develop this 2D software, the program will be soon extended to take into account the long wave effects and to solve the non steady radiative exchanges. The importance equations have not been used in this paper, but preliminary tests have shown that they will be useful to compute the power for each exitance source.

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