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Taking Advantage of Low Radiative Coupling in 3D Urban Models

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Abstract
We examine different techniques able to enhance the efficiency of urban simulations. Condensation techniques like superelements - well known in the finite element community - are good candidates to improve the heat transfers simulations inside the city. Three typical situations are examined. In the first one, concerned with the interaction of inside and outside scenes through windows, the two problems can be analyzed successively. The second situation presents a weak coupling of radiation between built settings. The last one refers to the full radiative budget of a city; it is used to illustrate the possibility of applying superelement techniques to the linear part of the problem. The conclusion is that these techniques are able to help enhancing the efficiency of the simulations.

1. Introduction

Cities have a huge variety of formats, but they also have specific geometric features that can be used to benefit data processing: the presence of buildings with their vertical windowed facades and flat or sloping roofs, delineating blocks separated by streets.

Once a 3D model of the city is completed, the first step is to visualize it. To do this, we use techniques of realistic rendering, which already make clear the radiative exchanges. This calculation, which follows the visible light, can be extended without additional cost to all short waves emitted by the Sun. This gives the solar gain on each urban surface, which is one of the thermal evaluation data. One can then extend this calculation to the waves emitted by the surfaces (thermographs), and so on, up to achieve a complete thermal calculation.

When considering the physical simulation, many computer graphics operations on the 3D model are highly relevant: level of detail, procedural methods. However, physics adds a second perspective on the 3D model, the consideration of couplings.

We shall present here three problems with three levels of coupling (zero, low, important) and show how physical considerations suggest a specific treatment of urban 3D models. In urban physics, the main existing codes for heat transfer calculation (Solene, [MG2001], Citysim, [ROB2011]) are based on the well referenced radiosity techniques [SP1994].

2. Form factors

To solve the system of radiosity equations, the heaviest part is the computation of the form factors of the matrix constituting the system. It is heavy because the number of coefficients is potentially very high (square of the number of elements) and because it involves the treatment of the visible surface detection.

2.1. General formula

The form factor (also called *view factor*, *angle factor* or *configuration factor*) is the basic ingredient of radiative heat transfer studies [SP1994]. It defines the fraction of the total power leaving patch A_i that is received by patch A_j. Its definition is purely geometric. The angles θ_i and θ_j relate to the direction of the vector connecting the differential elements with the vectors normal to these elements; r is the distance between the differential elements.

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} V(Y_i, Y_j) dA_i dA_j \quad (1.1)$$

Except in particular situations, it is not possible to compute (1.1) explicitly [HSM2011]. An additional difficulty appears in presence of obstructions represented in the above expression by the visibility function $V(X_i, Y_j)$. This function is equal to 0 or 1 according to the possible presence of an obstacle that does not allow seeing an element Y_i from an element Y_j.

2.2. Differential form factor

It is much easier to compute the differential form factor by removing the external integration that will be taken into account only in a second step to achieve the evaluation of the form factor, using, for instance, the Gauss rule. The differential form factor in a point surrounded by the element of area dS is given by:

$$F_{dS-A_j} = \int_{A_j} \frac{\cos\theta_i \ \cos\theta_j}{\pi r^2} V(Y_i, Y_j) \ dA_j \quad (1.2)$$

If the visibility function is everywhere equal to 1, the integration (1.2) performed on the full hemisphere is giving a form factor equal to 1. Spherical projections

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combined with Nusselt analogy provide an efficient solution of this problem [BMB2011].

3. Radiosity equations

In order to solve efficiently the interaction problem, it is usual to set up a discrete formulation derived from the global illumination equation by making the following assumption. The environment is a collection of a finite number N of small diffusively reflecting patches with uniform radiosity [SP1994].

This discrete formulation leads to a linear system of equations for which many algorithms are available.

$$\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix} + \begin{pmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1N} \\ \rho_2 F_{21} & \rho_2 F_{22} & & \vdots \\ \vdots & & & \vdots \\ \rho_N F_{N1} & \cdots & \cdots & \rho_N F_{NN} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} \quad (1.3)$$

$$B = E + GB$$

$$MB = E \;\; ; \; M_{ij} = \delta_{ij} - \rho_i F_{ij} \quad (1.4)$$

As seen in the construction of the G matrix, formed by the products of the form factors by the reflectances, it is a non symmetric system of linear equations (except if all the reflectances and the patch areas are equal), but the radiosity matrix M is diagonally dominant and well conditioned.

The components B_i are the radiosities or radiant fluxes per unit area, on patches i, E_i are the radiant exitances, ρ_i are the hemispherical diffuse reflectances, F_{ij} are the form factors between patches i and j.

Radiosity is the radiometric quantity that is best suited for quantifying the illumination in a diffuse scene. In practice, when there is one single problem to solve, iterative solutions are used, which require the treatment of only one line of the matrix per iteration.

If the process is dynamic, for instance due to the movement of the Sun and the varying configurations of the sky, it is convenient to mesh the whole sky and to give to each element of its mesh the emittance corresponding to the concerned situation.

At this step, it is more efficient to use the technique of combination of unitary right members. It means that all the components of a column of the right members are zero except one which is equal to 1. This system is solved for as many unitary right members as elements in the sky vault mesh, for instance, 145 in the Tregenza [TRE1987] dome and much more (Figure **1**) if necessary. The creation of this kind of dome is very simple because it is based on the two classical geographical coordinates (latitude and longitude). This choice facilitates the positioning and the navigation in the mesh [BB2012]. Its definition is given by the sequence of numbers of elements in each ring. From these data, it is easy to compute the partition of a disk into equal area elements. The geometric transformation between a hemisphere and its equal area projection allows projecting the disk elements on the sphere.

After solving the radiosity equations, it is sufficient to recombine the solutions for each particular situation for which it is possible to evaluate the right member. The consequence is that the computation of the radiosities is very cheap. Let us separate sky (index s) and city (index u) contributions in (1.3):

$$\begin{pmatrix} B_s \\ B_u \end{pmatrix} = \begin{pmatrix} E_s \\ E_u \end{pmatrix} + \begin{pmatrix} G_{ss} & G_{su} \\ G_{us} & G_{uu} \end{pmatrix} \begin{pmatrix} B_s \\ B_u \end{pmatrix} \quad (1.5)$$

The components of the matrix G are products of diffuse reflectances by form factors: $G_{ij} = \rho_i F_{ij}$. The sky is considered as a black surface, with zero reflectance, while the city is not producing shortwave radiation by itself. Therefore:

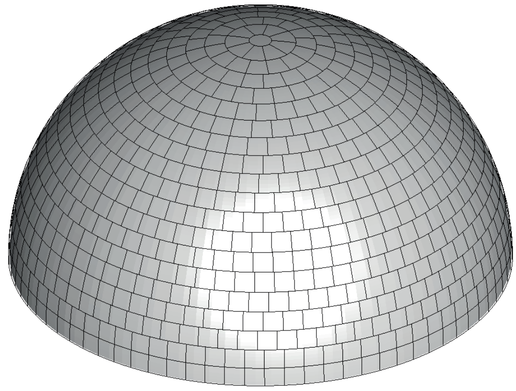

Figure 1: Hemispherical dome containing 1000 equal area cells

$$\begin{pmatrix} B_s \\ B_u \end{pmatrix} = \begin{pmatrix} E_s \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ G_{us} & G_{uu} \end{pmatrix} \begin{pmatrix} B_s \\ B_u \end{pmatrix} \quad (1.6)$$

Putting the solution of the first line in the second one, we obtain:

$$B_u = G_{us} B_s + G_{uu} B_u = G_{us} E_s + G_{uu} B_u \quad (1.7)$$

Grouping the radiosities from both sides:

$$M_{uu} B_u = G_{us} E_s \quad (1.8)$$

M_{uu} is the usual radiosity matrix whose dimension is the number of elements describing the model, G_{us} is a matrix containing the form factors related to the sky mesh (with as columns in this matrix as sky elements), and E_s is the right member (loads) corresponding to the emittances of the sky mesh elements (including the Sun).

The summation of all the coefficients of a line of G_{us} is giving the sky view factor of the element of the city mesh referred by this line multiplied by its reflectance.

Assuming that we replace the reflectance coefficients by one in matrices G_{us} and G_{uu}, we obtain the matrices F_{us} and F_{uu} of form factors. With respect to a city patch, the radiosities B_u are out coming radiations while the irradiances J_u are the incoming ones.

$$J_u = F_{uu} B_u + F_{us} E_s \quad (1.9)$$

If the city behaves as a black body, $B_u = 0$. In this case, expression (1.9) reflects the typical computations performed in the program Heliodon2^{TM} [BM2011]. When computing the diffuse illumination of a uniform sky, E_s reduces to a scalar and the matrix F_{us} has a single column containing the sky view factors of all the elements.

The radiosity matrix and the solution of the unit loads depend only on the geometries of the city and on the sky mesh. The combinations are taking into account the particular loadings, the geographic situation and the time evolution.

It is fundamental to note the linear dependence of the form factors evaluation cost regarding the sky dome mesh size. With respect to the computation cost, it is generally accepted that a number of right members reaching up to 10% of the number of variables is acceptable.

4. Multiscale problem

A town consists of two parts with very different scales: on one hand the outer surfaces (facades, roofs, streets, infrastructure elements), and on the other hand the inner surfaces (homes, offices). From the point of view of the irradiance, these two entities communicate through windows. Seen from outside, these behave in first approximation as black surfaces (their specular reflection is neglected). Seen from inside, they are also black surfaces (zero reflection coefficient) but they allow the entry of the radiations coming from sky and other buildings.

The processing of inside scenes runs in two steps. In the first one, the outside simulation, where all windows are considered as black surfaces, is performed. In the second step, the radiosity equations have the dimension of the number of internal patches but, for the loading, it is necessary to take into consideration all the exterior elements visible from inside.

Therefore, the two problems can be computed separately with adapted levels of discretization. It also means that the description of the external part of the windows involved in the interior scene has to achieve a sufficient level of details [BP 2011].

5. Weakly coupled system

The linearity of the system (1.8) implies the possibility of using the super element technique invented by aerospace engineers in the early 1960s.

Let us consider the situation of the presence in the city of numerous districts composed of closed patios with few connections with the other buildings.

It means that the variables which are weakly connected can be eliminated from the system of equations. We call them "condensed variables", while the remaining ones are called "conserved variables".

The unsymmetrical linear system of equations (1.8) can be decomposed in two parts, where index c corresponds to the condensed variables and r to the conserved ones.

$$\begin{bmatrix} M_{rr} & M_{rc} \\ M_{cr} & M_{cc} \end{bmatrix} \begin{bmatrix} B_r \\ B_c \end{bmatrix} = \begin{bmatrix} G_r^s \\ G_c^s \end{bmatrix} E^s \qquad (1.10)$$

The reduced (or condensed) system is obtained after the inversion of the sub-matrix M_{cc} which corresponds to the condensed radiosities B_c.

$$(M_{rr} + M_{rc}M_{cc}^{-1}M_{cr})B_r = (G_r^s - M_{rc}M_{cc}^{-1}G_c^s)E^s$$

(1.11)

In a second step, the condensed (or eliminated) radiosities can be evaluated.

$$B_c = M_{cc}^{-1}(G_c^s E^s - M_{cr}B_r) \qquad (1.12)$$

The main question is: how to define the superelements? We should try to find a (n_c x n_c) matrix M_{cc} with n_c as large as possible and matrices M_{cr} (n_c x n_r) and M_{rc} (n_r x n_c) with the dimension n_c as small as possible, which means that the two groups of radiosities B_c and B_r are weakly coupled.

In some opportunities, we can also imagine that the two groups are uncoupled, for example two patios with the same height and without communication. In this situation, the matrices M_{cr} and M_{rc} are filled with zeros. Any one member of the group c can see a group r element and vice versa.

Since the beginning of the finite element method, an extensive literature has been produced on matrix front and bandwidth minimization [WGS2009], [GUP2002], and handling of sparse matrices. All these techniques are designed to work directly on the matrices.

6. Longwave simulation

In this problem, several phenomena have to be taken into account. The most important aspect is the need to handle the full heat exchange problem including radiation, conduction and convection. It is also necessary to work with surface temperatures which makes the system highly non linear when radiative exchanges are present.

6.1. Stefan-Boltzmann's law

The Stefan-Boltzmann law states that the total energy (also known as irradiance or emissive power) radiated per unit surface of a black body per unit time is proportional (with the factor $\sigma = 5.6704\ 10^{-8} Wm^{-2}K^{-4}$) to the fourth power of the black body thermodynamic temperature.

Expressed as a function of the difference of temperatures $T_r - T_i = \Delta T$, with the reference temperature T_i, this law can be approximated so that, for instance, between 10 and 30°C, the approximate solution built around 20°C is giving a heat flux with less than 5% error Figure2.

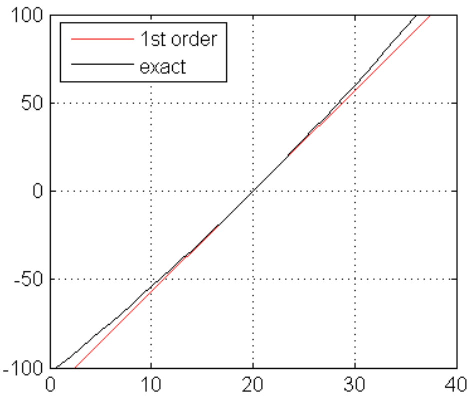

Figure 2: Transmitted power in Wm^{-2} according to Stefan-Boltzmann law and its 1st order approximation computed in the region of 20°C, (293.15 K)

$$Q = \sigma\left(T_r^4 - T_i^4\right)$$

$$Q = \sigma\ \Delta T\left(4T_i^3 + 6T_i^2\Delta T + 4T_i\Delta T^2 + \Delta T^3\right) \quad (1.13)$$

$$Q = 4\ \sigma T_i^3 \Delta T \qquad 1^{st}\text{order}$$

As shown in Figure 3, the flux between two infinite walls at temperatures differing from one degree, (1.14), is reaching 6 Wm^{-2} at 297 K (24 °C).

$$Q = \sigma\left(4T^3 + 6T^2 + 4T + 1\right) \quad \left(Wm^{-2}\right) \qquad (1.14)$$

Between a source at 279 K and a receptor at 0 K, the flux is equal to 343.6 Wm^{-2}. Note that the average Sun irradiance [BEC2012] at the top atmosphere (or without atmosphere on the ground) is of 342 Wm^{-2}. The temperature of 279 K should be the Earth's temperature originating longwave radiations able to balance Sun's radiations in the lack of atmosphere.

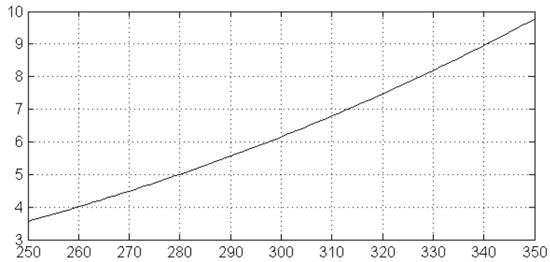

Figure 3: Heat flux in Wm^{-2} between two infinite walls whose temperatures differ from 1 degree

6.2. The finite element model

The finite element method is illustrated with a simple example [HSM2011]. In a 1D radiating fin model, the local equilibrium links terms of conduction (*k*: thermal conductivity) and heat storage (ρc, density and specific heat) in the solid (*Adx*: volume, cross section by differential length), and terms of convection (*h*: convective coefficient) and radiative exchanges ($\sigma\varepsilon$: Stefan-Boltzmann's constant, emissivity of a surface) throughout the lateral skin (*Pdx*: lateral surface by differential length) while T_e is a reference temperature in the fluid surrounding the fin.

$$(k\frac{\partial^2 T}{\partial x^2}dx + \rho c\frac{\partial T}{\partial t})Adx = (h\left[T - T_e\right] + \sigma\varepsilon\left[T^4 - T_e^4\right])Pdx \tag{1.15}$$

This set of transient equations is linear with respect to conduction and convection but not radiation.

The finite element method is a numerical technique for finding approximate solution of this kind of equation.

Using this method to compute the heat fluxes at the city level needs to discretize the solid objects with a congruent finite element mesh. This mesh is sustaining the finite elements intended to simulate the conduction process and the accumulation of heat produced by the shortwave irradiation [EEK2012].

The buildings are also exchanging heat with the ambient air by convection: receiving or returning it, depending on the difference of temperature.

Finally, heat is returned to the atmosphere by longwave irradiation.

For radiative exchanges occurring between buildings, however, the temperatures are often differing with a small gap and, as shown in Figure 2, the Stefan-Boltzmann law (1.13) can be replaced by its first order approximation.

In clear sky conditions, due to higher differences of temperatures (about -50°C at zenith up to 50°C or more on the ground), the heat exchanges between sky and city are non linear. However, for cloudy skies, the difference can drop drastically [VM2010].

It is then very interesting to condense in a super element the linear part of the model and to iterate on the degrees of freedom corresponding to the elements of the city participating at high level to the heat fluxes going to the sky, i.e. by selecting only the roofs or other elements of the scene. For this problem, procedural methods will help to organize the data [BP 2011].

7. Conclusion

With three examples, we have shown the interest to find techniques in order to alleviate the heavy simulation processes. The main objective is to allow repeating the calculation, either during the design process to compare different configurations, either in the final analysis of the city behavior on a long period of time.

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