FICUP
An International Conference on
Urban Physics
B. Beckers, T. Pico, S. Jimenez (Eds.)
Quito – Galápagos, Ecuador, 26 – 30 September 2016

Fast and accurate view factor generation

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Keywords: View Factor, Ray Tracing, Uniform Mesh, Radiative Heat Transfer, Stratified Sampling.

Abstract. *This document explains how to mesh the hemisphere with equal view factor elements. The main characteristic of the method is the definition of elements delimited by the two classical spherical coordinates (polar and azimuth angles) similar to the geographical longitude and latitude. This choice is very convenient to identify the localization of the elements on the sphere; it also simplifies a lot the determination of rays for either deterministic or stratified sampled Monte Carlo ray tracing. The generation of the mesh is very fast and consequently well suited for ray tracing methods. The quality of the set of rays spatially very well distributed is a fundamental element of the whole process reliability.*

1 Introduction

The main radiative phenomena considered in urban physics are: light, sound and heat. In thermal radiation, we must distinguish between exchanges that occur in short wavelengths (including visible light) and those that take place in the long wavelengths [Beckers 2011]. The objects of the urban scene only emit in long wavelengths, with an intensity that is proportional to the fourth power of their temperature. Thermal loads due to Sun are totally provided in shortwave, and their interaction with the city surfaces is independent of the temperatures.

The fundamental differences between these problems come from the wave propagation behavior and the human perception: light is considered instantaneous, sound is perceived delayed, and heat involves inertia [Beckers 2014a].

To solve radiative problems, we distinguish two completely different approaches. The first one is using some kind of mesh generated in CAD systems (typically the wide used “stl” files), finite element or radiosity methods [Beckers 2016]; the second deals only with discretized sources and uses ray tracing techniques, typically in the frame of Monte Carlo methods.

In the first approach, the problem is based on the discretization of the objects into elements or patches that will be used to model the scene and simulate the physical behavior. The basic ingredients are the view factors. These are purely geometrical parameters that describe how objects are seen from each other. They can be computed by algebraic or Monte Carlo ray tracing methods.

The paper is mainly based on [Beckers 2012], where the idea of using spherical equal area cells was introduced for the first time. The concept of coverage index, initially introduced in [Tregenza 1987] and enhanced in [Beckers 2014b], is actually giving valuable information on the cells aspect ratios. The geometric backgrounds of the method are fully developed in [Beckers 2014a].

2 View Factor

The *view factor* (also called *form factor*) is the basic element of the radiative studies [Beckers 2014a, Beckers 2012, Sillion 1994]. It defines the fraction of the total power leaving patch A_i that is received by patch A_j. Its definition is purely geometric. The angles θ_i and θ_j relate to the direction of the vector connecting the differential elements with the vectors normal to these elements; r_{ij} is the distance between the differential elements.

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r_{ij}^2} V(y_i, y_j)\, dA_i dA_j \tag{1}$$

Except in particular situations [Howel 2010], it is not possible to compute the view factors explicitly. An additional difficulty appears in presence of obstructions represented in the above expression by the visibility function $V(y_i, y_j)$. This function is equal to 0 or 1 according to the possible presence of an obstacle that does not allow seeing an element y_j from an element y_i.

It is much easier to compute the differential view factor by removing the external integration that will be taken into account thereafter in order to achieve the evaluation of the view factor, using, for instance, Gaussian quadrature rule. The differential view factor in a point y_i surrounded by the element area dA_i is given by:

$$F_{dA_i - A_j} = \int_{A_j} \frac{\cos\theta_i \ \cos\theta_j}{\pi r_{ij}^2} V(y_i, y_j)\, dA_j \tag{2}$$

This expression can be interpreted as the result of two successive operations known as Nusselt analogy, where we will momentarily disregard the visibility term $V\ (y_i,\ y_j)$ not required for the explanation:

1. The element is projected on the unit hemisphere centered on the point y_i. This step is represented by the factor $\cos\theta_j / r_{ij}^2$ of relation (2). The solid angle completed by the element dA_j, which is also the area of the spherical polygon built from the same element, is given by

$$d\omega_j = \frac{\cos\theta_j}{r_{ij}^2} dA_j \tag{3}$$

2. The spherical polygon is orthogonally projected on the base plane dA_i. This projection corresponds to the term cos θ_i of relation (2), which is now transformed into:

$$F_{dA_i - A_j} = \int_{\Omega_j} \frac{\cos\theta_i}{\pi} d\omega_j \tag{4}$$

The term Ω_j represents the solid angle or the spherical polygon area subtended by A_j. The view factor is expressed in percents (projected area over unit disk area by 100).

3 Computing the View Factor

The view factor can be calculated principally in two ways: algebraic methods or ray tracing methods. In the first situation, the geometry of the scene has to be modeled. In the second case, we do not need the deep description of the scene: it is sufficient to give a set of simple patches or triangles like in the "stl" format, which comes from the stereolithography CAD software and is widely used for rapid prototyping, 3D printing and computer-aided manufacturing.

So, the first way to calculate the differential view factor, shown in relations (3) and (4), is to project it onto the hemisphere defined at the concerned point and then to project the spherical polygon orthogonally on the plane tangent to the surface (the disk which is the base of the hemisphere). This projection is compared to the area of the disk. The calculation method is in principle easy to implement. Both steps are easy to perform for any shape that can be decomposed in small line segments. This procedure is applicable for any parameterized shape.

The foundation of the first step is a central projection on a unit sphere centered at origin, which consists in dividing the positions by their modules:

$$P' = \frac{P}{|P|} \tag{5}$$

The second step, which is the orthogonal projection of P', is straightforward provided we are working in axes defined with respect to the projection plane (normal vector n).

$$P'' = P' - (P' \cdot n)n \tag{6}$$

Let us start with the computation of the view factor of a polyline $P_{i-i}\ P_i\ P_{i+i}\ldots$which is not necessarily in a plane. It is shown in blue lines in Figure 1. To compute from point O the view factor of this figure, we have to proceed in two steps. First, we project it on the unit sphere represented in the figure by its base and two orthogonal semi-meridians, respectively in the plane $x = 0$ and $y = 0$.

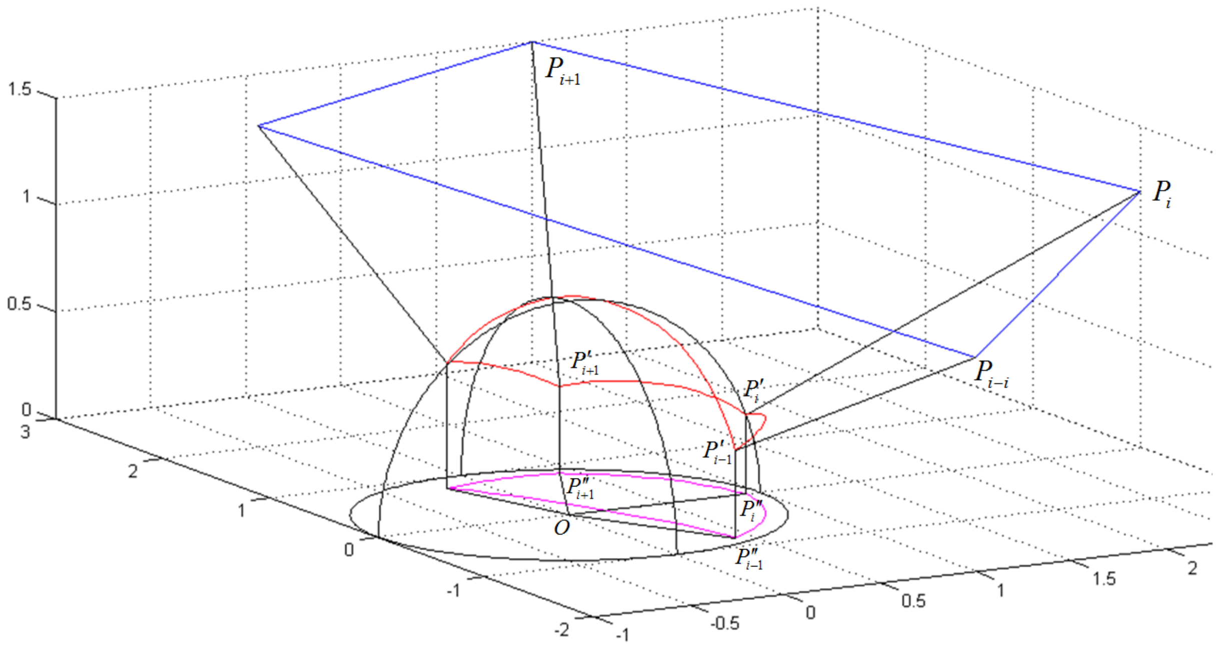

Figure 1: View factor: Point to patch

The spherical projection drawn in red is composed of great circle arcs. In the figure, P'_{i-1}, P'_i and P'_{i+1} are the spherical projections (5) of P_{i-1}, P_i and P_{i+1}. In a second step, we build the orthogonal projection of the spherical polygon on the base of the hemisphere: plane $z = 0$. The circular arcs are transformed into elliptical ones (with the two limiting cases of straight lines or circular arcs). In the figure, P''_{i-1} and P''_i are the orthogonal projections (6) of P'_{i-1} and P'_i.

To compute the view factor, we have first to define the unit vectors f_i normal to the faces of the spherical pyramid $OP'_{i-1}\ P'_i\ P'_{i+1}\ldots$ where OP'_{i-1}, OP'_i,… are unit vectors computed from the apex O to the vertices of the studied contour $P_{i-i}\ P_i\ P_{i+i}\ldots$ The vertices sequence of the pyramid base is defined in such a way that the spherical polygon representing its projection on the sphere is always situated on the left side of its boundary composed of great circles segments.

$$f_i = \left(\frac{OP'_{i-1} \times OP'_i}{\left| OP'_{i-1} \times OP'_i \right|} \right) \tag{7}$$

The length l_i of the circular segment $P'_{i-1}\ P'_i$ is given by:

$$l_i = \arcsin\ \left(\left| OP'_{i-1} \times OP'_i \right| \right) \tag{8}$$

It is always positive because the arc length is greater than zero and less than π. Because the area of a unit disk sector of angle α is equal to α/2, the arc length of the spherical pyramid face $OP'_{i-1}\ P'_i$ is equal to twice its area. The orthogonal projection a_i of the face area on the base plane with normal vector n is then given by:

$$a_i = \frac{l_i}{2}(f_i\ .\ n) \tag{9}$$

The vector n is normal to the surface supporting dS and on which we calculate the view factor. As defined in (7), the vectors f_i are normal to the faces of the pyramid: $OP_{i-i}P_i$, OP_iP_{i+1} …The dot products of (9) are multiplied by the quantities l_i, equal to the angles of the faces of the pyramid at the apex O. This expression can be positive or negative, depending on its orientation given by the dot product.

If we add algebraically the expressions (9) for all the contour segments, we obtain the area of the orthogonal projection $P''_{i-1}\ P''_i\ P''_{i+1}$ …of the spherical polygon, which must be divided by π (area of the base) to obtain the relative area:

$$F_{dS-P_j} = \frac{1}{\pi}\sum_i a_i = \frac{1}{2\pi}\sum_i l_i\left(f_i \cdot n\right) \tag{10}$$

For a shape $P_{i-i}\ P_i\ P_{i+i}$ …, the formula is giving a result that depends only on the accuracy of its evaluation. This shape can be as simple as a polygon or it can be extracted from the outline of a solid and expressed as a polyline. The precision also depends on the precision of the computation of the obstructions. In complex situations, these computations can be very heavy.

If the patches do not cover the full hemisphere, the complement to 1 of the sum of their view factors is called sky view factor (closure property of the view factors). The sky view factor is linked to the visible part of the vault of heaven; it is often used as design parameter in architectural applications. When the skyline is available, (10) can provide an easy and fast method to compute the sky view factor.

4 Meshing the Hemisphere

Before considering the second method used for computing the view factors, we have first to consider the spherical support used to generate the rays for the casting process. There are several methods to mesh a sphere: in the first one, it is covered with spherical polygons that are figures of the sphere delimited by great circles. In practice, these structures are based on some of the five regular spherical polygons.

In another one, we build elements bounded by segments of parallels and meridians. The choice of this kind of mesh is justified by the fact that the spherical coordinates based on polar and azimuth angles (where the polar angle may be called co-latitude, zenith angle, normal angle, or inclination angle) or the geographical coordinates are widely used to describe the sphere. A direct advantage of this choice is that the azimuthal projections centered on the poles of these elements are figures of the circle bounded by arcs of concentric circles and radii segments [Beckers 2014b, Leopardi 2006]. For these reasons, it is our preferred meshing method.

But before addressing the problem of the hemisphere, we first examine how to define equal area cells within a disk. The full disk is divided into a central one surrounded by concentric rings, each one containing a certain number of cells. For a mesh where all elements have the same area, one realizes immediately that the sequence of cells differs on the different rings.

Let assume that N equal cells have to be defined in a unit disk. Starting from a central disk composed of a single cell and whose radius is equal to $r_1 = 1/\sqrt{N}$, we easily perform the computation in the ring surrounding it. This disk is composed of n cells, so that the disk that is the sum of the inner disc and this one contains ($k_2 = k_1 + n$) cells (or $k_{i+1} = k_i + n$). The radius of this disc is given by $r_{i+1} = r_i\sqrt{k_{i+1}}$. The number of cells added to each ring is arbitrary, provided that the total amount of cells does not exceed the value N.

As the filling sequence of the successive disks is arbitrary, we deduce that it is possible to impose at each step an additional condition, for example imposing the aspect ratio of the cells, either in the ring to be inserted on the disk (Figure 2), or on the hemisphere (Figure 3). This procedure only needs a few statements in Matlab and gives the sequence of cells in the different rings, from the spherical cap on the top of the dome to its base. For the example of 100 imposed cells of Figure 2, we have the non optimized sequence:

$$S = \begin{bmatrix} 1 & 8 & 22 & 42 & 68 & 100 \end{bmatrix} \tag{11}$$

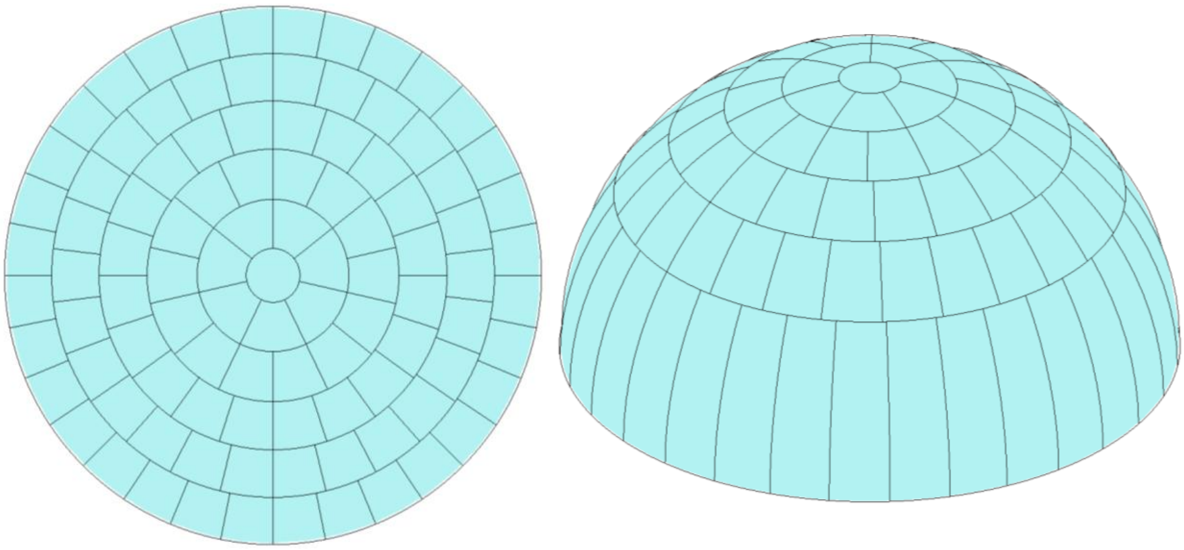

Figure 2: 2D and 3D views of 100 cells with equal areas and aspect ratio equal to 1 on the disk

In the optimized case of Figure 3, obtained with the functions developed in [Beckers 2016b], we obtain the sequence:

$$S = \begin{bmatrix} 1 & 8 & 22 & 40 & 62 & 84 & 100 \end{bmatrix} \tag{12}$$

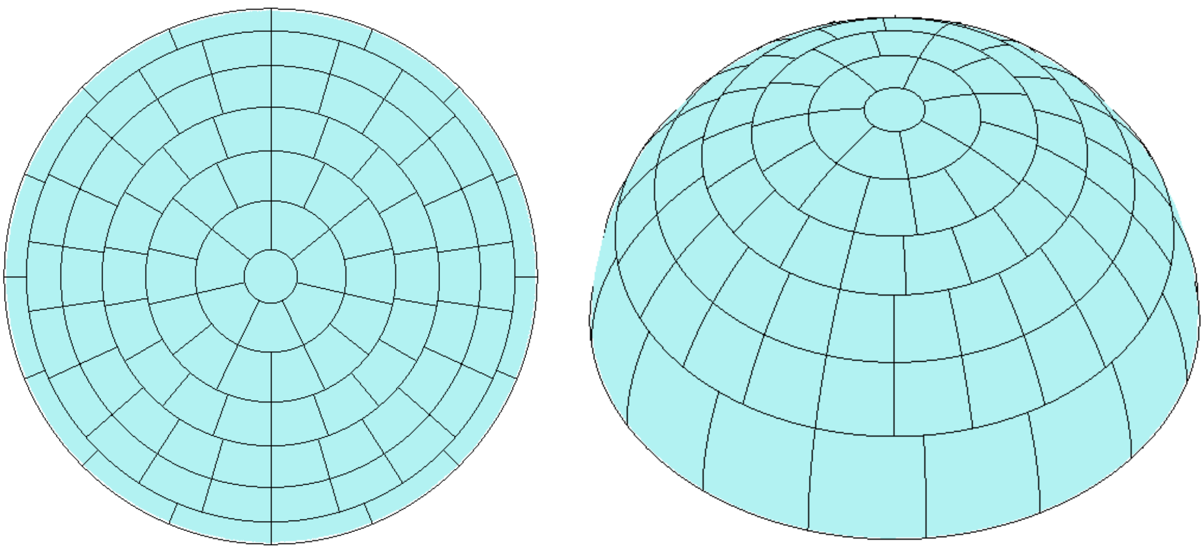

Figure 3: 100 cells with same areas on the disk and aspect ratio equal to 1 on the hemisphere

Once the sequence of cells is defined on the disk, it is easy to use an inverse azimuthal projection to obtain the mesh on the sphere. In the case of azimuthal orthogonal projection, the relationship between the polar angle θ on the unit hemisphere measured in radians and the radius in the projection is:

$$r = \sin\theta \qquad (13)$$

On the left side of Figure 2, we see the orthogonal projection of the hemisphere on its base. Here, both the areas and the aspect ratios of the projection are equal. The drawback of this choice is the important distortion of the cells close to the base of the hemisphere. In Figure 3, the areas of the projection are required to be equal while the aspect ratios are required to be equal to one on the hemisphere. The important distortion of the cells close to the base is now removed.

When we significantly increase the number of cells, we observe first that the processing time needed to generate the sequence of cells is negligible and secondly that the main difference between optimized (Figure 4, left) and non optimized (Figure 4, right) situations is occurring mainly close to the base.

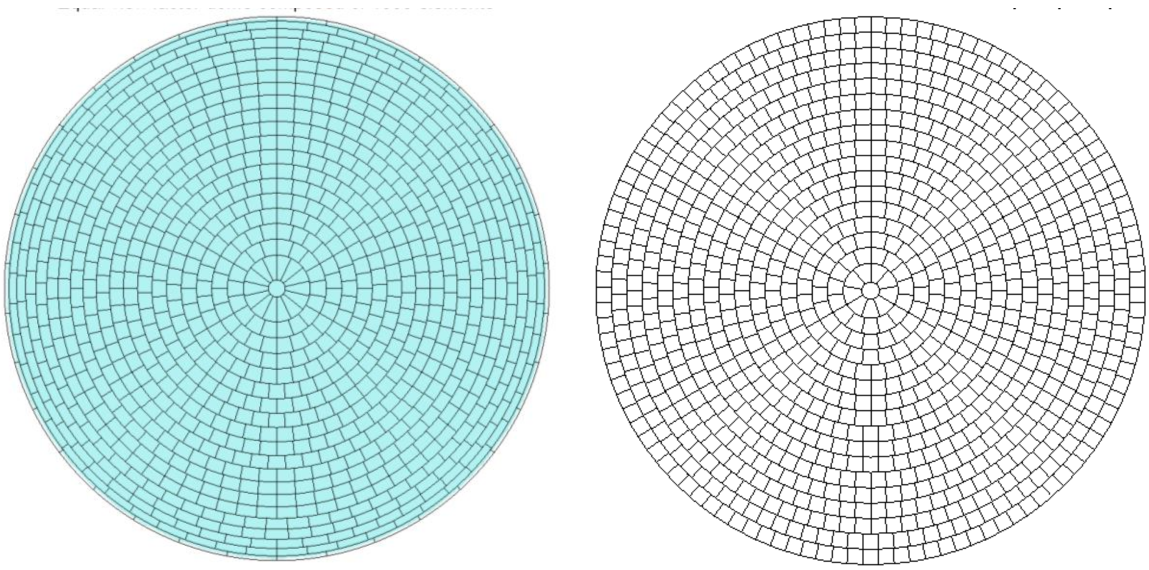

Figure 4: Comparison of the 2 solutions for a generation of 1000 cells

5 Generating rays

After the generation of equal view factor cells, it is possible to generate rays that will allow computing view factors of the scene elements. The rays are generated, for instance from the origin to each cell and traced to the scene, and the number of collisions with the elements is accounted. The view factor of an element is the ratio between the number of impacting rays and the total number of traced rays. If the number of traced rays is sufficient, the result tends to the exact solution [Vujicic 2006].

The first method used to define the rays is deterministic, for instance, the rays pass through the center of each cell. It is the situation shown in the orthogonal projections Figure 5 & Figure 6 of the optimized cell sequence [1 15 32 54 80 107 133 151].

In a non optimized sequence [1 9 22 41 64 91 120 151], we observe the bad aspect ratio of the lower ring (Figure 7); it is confirmed by the diagram of Figure 8 showing the relative coverage index in each ring. This index is defined as the ratio of the area of the greatest inscribed circle and the cell area, compared to the same ratio computed in a plane square and equal to $\pi/4$ [Beckers 2014b]. It also appears clearly that the density of points is lower in the bottom of the dome (Figure 6). The same is occurring for the random rays of Figure 10.

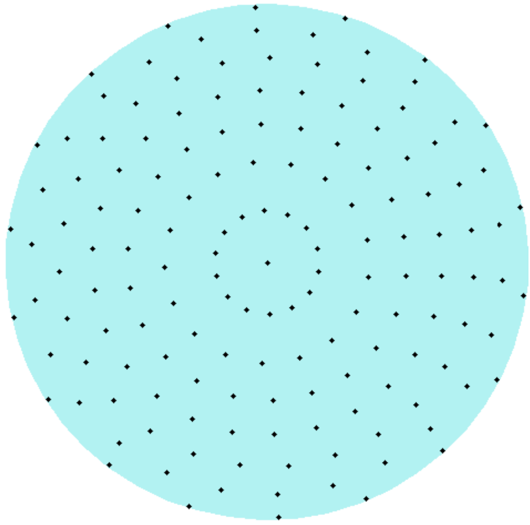

Figure 5: Deterministic 151 cells centers

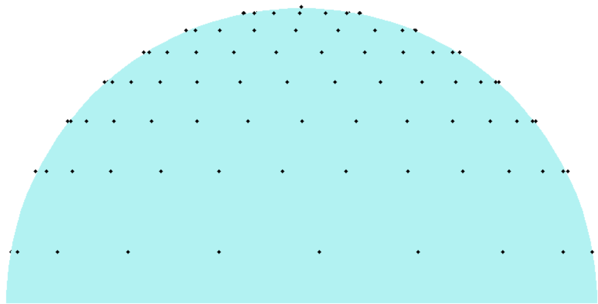

Figure 6: Side view of the dome composed of 151 rays generated from equal view factor cells

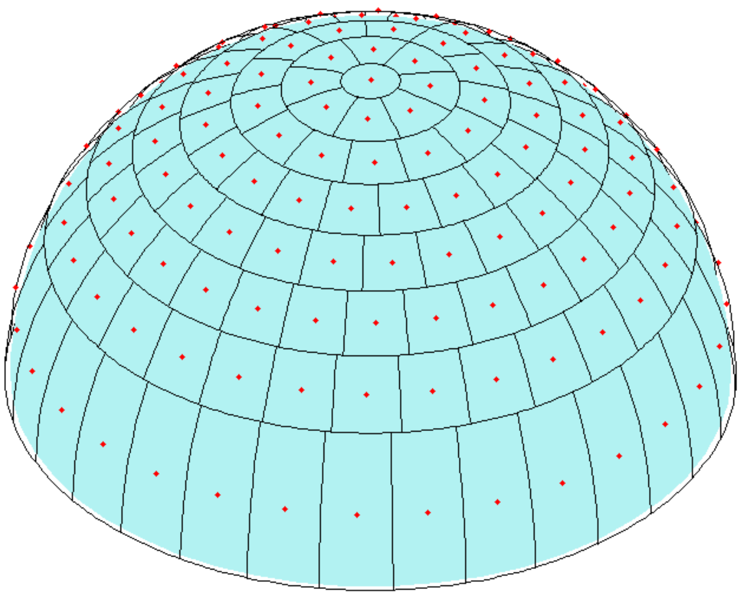

Figure 7: Mesh and deterministic rays for the non optimized 151 cells dome

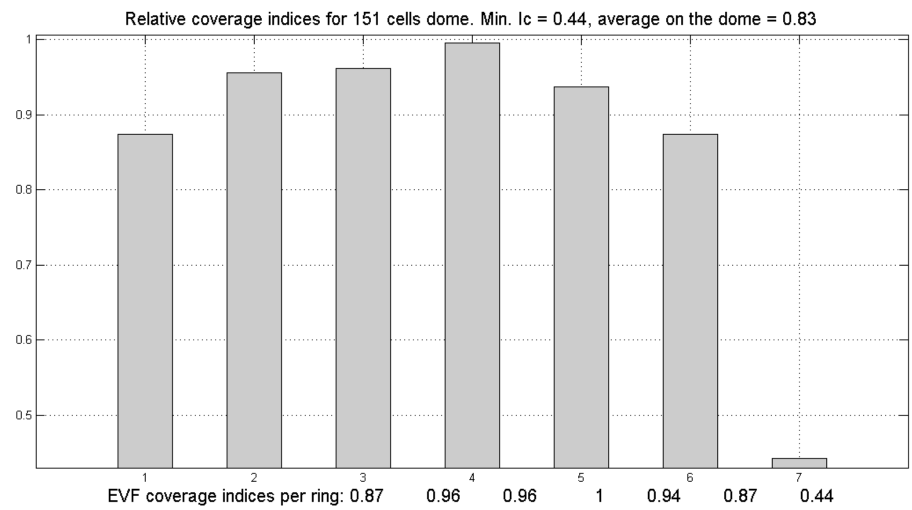

Figure 8: Equal view factor (EVF), 151 cells mesh without cells aspect ratio optimization

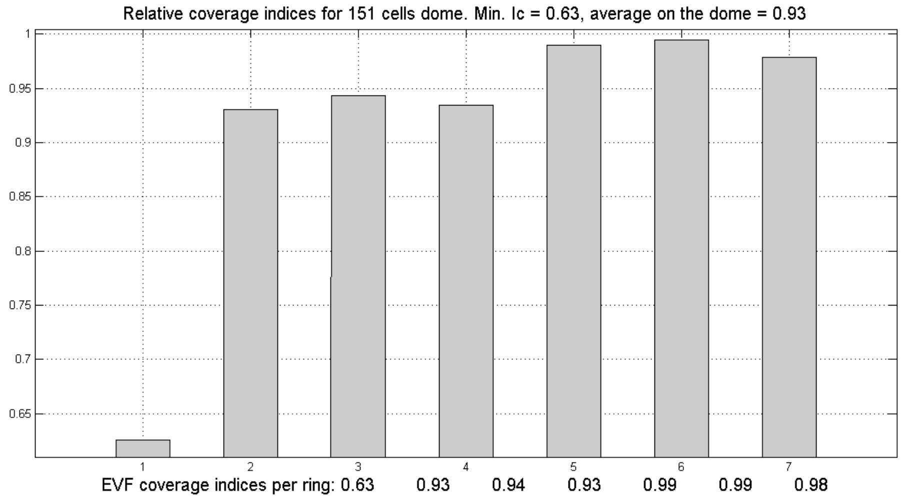

Figure 9: Equal view factor (EVF) optimized 151 cells mesh

In the optimized mesh where the cells aspects ratios on the sphere are close to 1, we obtain the new cells sequence [1 15 32 54 80 107 133 151] and the coverage indices of Figure 9. We observe that the worse coverage index occurs in the ring close to the top of the dome while it occurs in the bottom ring of the non optimized sequence. Anyway, the optimized sequence is better both for the minimum value and for the average.

In the second ray tracing method, the position in each cell is defined randomly. Because all the cells are defined between two latitudes and two longitudes, this procedure is very reliable and easy to implement. This method pertains to the category of stratified sampled Monte Carlo methods. An example of this kind of ray distribution is shown on a side view of a dome in Figure 10. It appears clearly that the density of points is lower close to the base of the dome, which reflects the behavior of importance sampling methods.

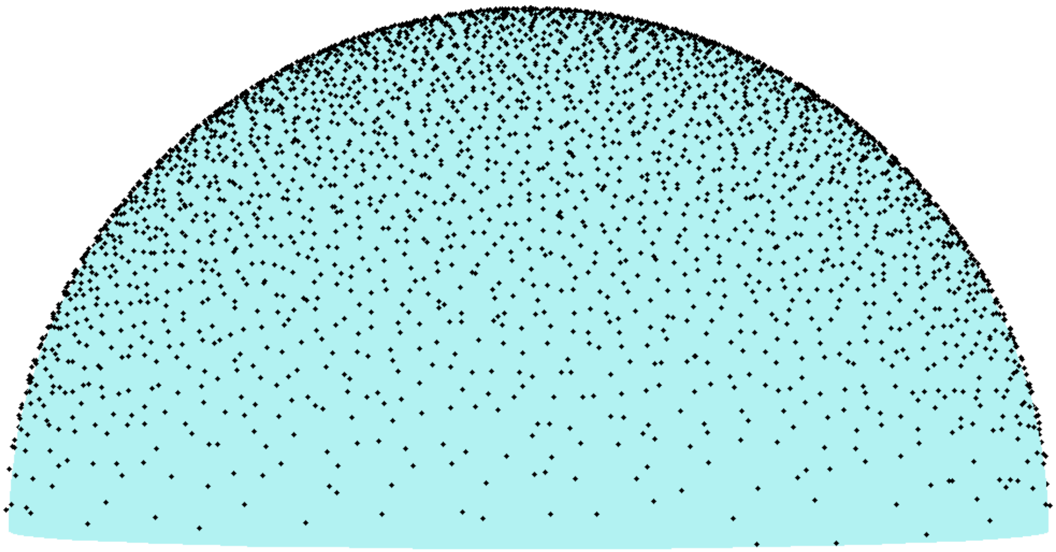

Figure 10: Side view of a dome composed of 5000 random rays

The proposed method is also the most convenient one to generate uniform equal solid angle rays on the sphere. In this case, as proposed in [Beckers 2012], it is similar to that of [Leopardi 2006], but according to the performed comparative tests, we feel that it is faster, because it is using a pure algebraic procedure.

6 Conclusion

Two methods are proposed for computing the view factors. The first one, often called Lambert method [Beckers 2014a], uses an explicit formulation of the point to patch view factor. It is very efficient and exact in the case of lack of obstacle between the point and the patch. The second one is based on an original method of mesh generation on the sphere or the hemisphere. This kind of mesh allows using both importance and stratified sampling in Monte Carlo ray tracing methods. It provides an efficient method to compute the view factors in complex urban environments because due to its geometrical simplicity, it is naturally well suited to deal with complex spatial configurations.

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