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A robust smoothed voxel representation for the generation of finite element models for computational urban physics

Alain Rassineux[1], **Benoit Beckers**[1]

[1] Sorbonne universités, Université de Technologie de Compiègne, CNRS

Laboratoire Roberval UMR 7337 CNRS/UTC 7337, Centre de Recherche de Royallieu, BP 20529, F60205 Compiègne
e-mail: alain.rassineux@utc.fr

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Abstract. *A voxel based methodology to create high quality finite element meshes of large city models for urban computational physics is proposed. Initial data are limited to 3D points cloud or STL files. A voxel grid which provides a rough representation of the outer (visible) surfaces of geometry is first created. The voxel model is then smoothed by a subdivision technique which provides a better representation of the normal vector to the surface than the voxel model. The points smoothed models are thereafter projected on the initial data. The ground, building blocks and the air are identified as volumes and their corresponding closed conforming surface mesh is created. Examples of complex reconstructions are presented.*

1. Introduction

The work proposed here is mostly based on the generation of finite element meshes of complex urban models. When dealing with complex geometry assembly, the major time of the finite element analysis (FEA) is spent on the creation of the mesh model. Mesh generation based on CAD for computational mechanics for instance has been widely investigated [De Cougny 1996]. The model, generally represented by parametric surfaces such as NURBS is meshed patch by patch in a suitable parametric space. Each volume must be represented by a closed BREP which in addition must allow to create a conforming mesh at interfaces between patches. The CAD model must be "cleaned" manually what represents a tedious and time consuming task. In addition, when the number of volumes increases, the cost of the creation of a geometry suitable for meshing becomes prohibitive. Moreover, the CAD model often contains details which are not necessary for the analysis. This may even impede the convergence of the analysis or make the cost of the computation prohibitive. When dealing with explicit analysis for instance, the time step is linked to the smallest element which may be located on a detail of the CAD which has no influence on the FEA. A large amount of time is often spent to remove details from the CAD before creating a relevant mesh.

A second class of techniques which does not use a surface model consists in using a polyhedral representation of the object such as a STL mesh. Most CAD systems can export a surface geometry into a stereolithography (STL) surface mesh file. STL representation provides a good approximation of the surface model into triangles but a very poor mesh for finite element analysis purposes and the mesh must be enhanced [Hassan 2004]. A number of authors have proposed remeshing procedures based on local optimization procedures [Chappuis 2004]. These techniques involve extracting from the surface mesh sets of triangles sharing the same node or the same edge and then remeshing the outer contour to a higher criterion (size or shape). These procedures can be used to refine or coarsen the mesh or more generally to enhance mesh quality with respect to a preset criterion (shape, size, anisotropy). The experience shows that the efficiency of these procedures is considerably reduced when the mesh is highly distorted. Moreover, the initial data mesh must still be conforming; free of intersection, overlappings what remains a severe constraint when dealing with complex heterogeneous urban models. A model composed on hundreds or thousands of buildings, including the ground can hardly be represented with these previous techniques, what explains the use of other techniques in the context of urban modelling such as the use of procedural modeling [Müller 2006], [Lienhard 2014] which enables to create a wide range of architectural detailed model or model reconstruction based on 3D-points clouds [Berger 2016] obtained from aerial data for instance

Lafarge and Mallet [Lafarge 2012] have proposed a survey on a number of these techniques together with an approach based on an energy minimization criterion to reconstruct large scale city models from 3D-points clouds. Geometrical features usually met in urban architecture (façade, roof) such as planes, cylinders, sphere and cones are identified. Complex environments at different scales are rebuilt with accuracy as often required by architects. Most of these techniques allow a very realistic representation of the geometry what may not be the major requirement when dealing with FEA as mentioned previously.

Another class of technique frequently used to create an approximation of complex FE models consists in using a grid [Hello 2014] as a representation model, in most cases when there is no other solution! This grid can be composed of voxels of regular size or be adaptive. In the last case, an octree structure is built [Vo 2015]. The surface to represent can be given by the outer faces of the grid what provides a shagged effect. A smoother surface representation can also be obtained by marching cubes or at a higher cost a level set technique.

A number of simulations such as convection-diffusion can be carried out with a finite difference solver which only requires a grid, usually regular to make the generation easier. However, when dealing with elliptic problems, the Finite Element Method (FEM) has the ability to take into account natural boundary conditions included in the weak form and is therefore much preferred not to mention non linear problems for which FEM must be preferred. In addition, adaptation can be obtained at a much lower cost with FEA. We shall focus here on a technique to generate meshes for finite element analysis. In this context, "Quality" of mesh and especially finite element has been widely discussed [Zienkiewicz 2013]. From a computational point of view, the quality of the mesh is ideally linked to the result of the analysis compared to experimental results provided that these results exist. The accuracy at which the geometry must be described is indeed highly dependent on the physical phenomenon to study and at what scale the phenomenon is studied. For instance, the study of the integrity of a building structure would indeed require an accurate representation of the components on which the loads are applied. The study of direct solar irradiation [Vermeulen 2015] or the calculation of the surface albedo or the compactness of a city may need a much coarser representation of the facade or rooves and a number of details may be eliminated [Beckers 2013]. The choice of the element type in FEA is also an important issue and depending on the simulation, a shell or volume mesh can be chosen for the buildings, ground, or air.

We propose a robust voxel based reconstruction technique dedicated to large scale city models which provides at the same time a high quality finite element mesh for shell or volume simulation while describing all complex boundaries between model components. The approach may use STL file of any quality or 3D point clouds. The technique allow to reconstruct the outer (visible) surfaces of the buildings while removing from the model details which, for a number of applications and at a chosen scale, may not alter the quality of the analysis. Sets composed of adjacent buildings are identified and a closed mesh of each set is created. The technique allows to generate a conforming meshes of both ground and space around the buildings. When the geometry is decomposed into meshable pieces, the same mesh must be created at the interface of shared surfaces in order to create a conforming assembly mesh. This major requirement of traditional FEM is a bottleneck when dealing with large models with noise, gaps, overlap which often require to reconstruct the whole model. Facades and rooves can also be identified. As each component is represented by a closed mesh, all volumes can be easily calculated. Once a mesh is obtained, usual mesh coarsening can be thereafter applied in order to reduce the computational cost if needed. The advantages and drawbacks of the simplified model are discussed and illustrated.

The different steps of the methodology are now described.

2 Voxelisation

In a first step, the initial data (triangles or points) are enclosed in a regularly spaced three-dimensional voxel grid. Voxel size is a user parameter. The resolution of the box in the 3 directions X, Y, Z of the Cartesian coordinate system is given by the triplet $(n_{x,}n_{y,}n_{z,})$. If a STL file is provided, triangles are discretized using the finite element shape functions so that the maximal space between neighboring points does not exceed 10% of the chosen voxel size. The input data is therefore in all cases a set a points. Point coordinates are mapped in a space of dimension $(n_{x,}n_{y,}n_{z,})$ and origin $(0,0,0)$ denoted as $(0, X, Y, Z)$. Voxels can be easily numbered from 0 to $n_x n_y n_z - 1$.

The number of a voxel of origin (i, j, k) denoted as $V(i, j, k)$ is given by equation (1).

$$i + n_x \times j + n_x \times n_y \times k \tag{1}$$

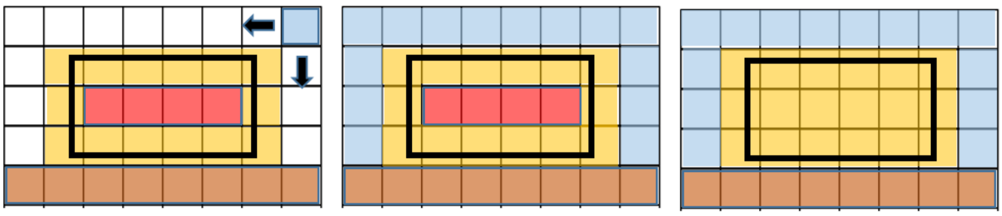
The 6 neighboring voxels in the 3 directions X, Y, Z (except on the boarder of the grid) are respectively

$$V(i-1, j, k), V(i+1, j, k), V(i, j-1, k), V(i, j+1, k), V(i, j, k-1), V(i, j, k+1)$$

A point of coordinates (x, y, z) in $(0, X, Y, Z)$ is located in the voxel $V(c(x), c(y), c(z))$ where $c(x)$ denotes the smallest integral value that is not less than x.

The only requirement of the technique is to provide a representation of the ground or in other words to give a segmentation of the points into ground and other points to be classified.

Voxels are sorted into 3 categories denoted as ground, building and air voxels. Ground points are first inserted in the grid and ground voxels are defined. The other points are assumed to be building voxels and are also inserted. Indeed, a voxel may contain both ground and building points and priority is given to the ground. As only surfaces points are provided, voxels inside a building with no interior points are not yet classified as building voxels.



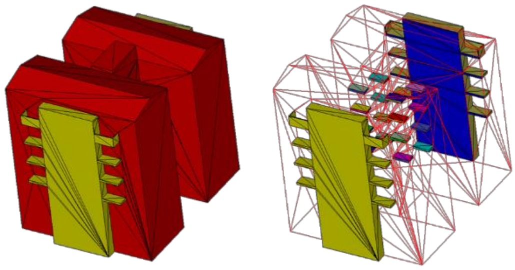
(a) (b) (c)

Figure 1. Sorting the voxels: ground, building, air

The sorting process is illustrated in figures 1. The points of the outer surface of a building are represented by a rectangle. In figure 1a, voxels containing ground points have been determined as well as voxels containing building points in a second step.

A prior check is made in order to detect empty cells which may be between the ground and the building. These cells are assigned to buildings. If no ground voxel is found under a building voxel, the cell is reassigned to a ground cell. Voxels (red on the figure) inside the building containing no points have no assignation yet. An empty voxel is chosen on the top of the grid and an advancing front algorithm based on face adjacency easily obtained is applied to fill all cells around the buildings which have not been assigned yet. The result is shown in figure 1b. In practice, a row of empty voxel is added on the top on the grid in order to create a unique volume of air around the buildings. Voxels which have not been assigned are defined as building voxels in a final step. The results is shown in figure 1c.

As represented in figure 2b, STL models may be composed of a high number of simple volumetric shapes which intersects and the model may also contain inner surfaces. The above technique allows to clean easily inner or even spurious surfaces.



(a) (b)

Figures 2. STL representation of a building. (a) outer surfaces (b) inner surfaces to remove

3 Creation of building blocks

All cells have been assigned to ground, building or air. We suppose that both ground and air are composed of a single volume.

3.1 Volume determination

An advancing front technique is used again to determine all different volumes (buildings). The algorithm widely used can be easily described as follows.

```
For all building cells (voxels)
{
    If the cell has already been assigned to a volume choose another cell
    Otherwise the front is initially constituted of this unique cell
    While the front is not empty
    {
        Determine for each face of the cell all other adjacent building voxels
        Add these adjacent cells to the front if not yet inside
        Remove the cell from the front
    }
    Increase the number of volumes
}
```

At the end of the process, volumes constituted by a list of cube are created.

3.2 Surface of the building blocks

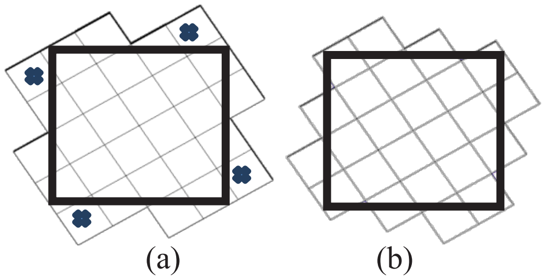
The maximum height, ground surface of a building, volume of each volume can be easily determined. Depending on the voxel size, different thresholds can be given to eliminate “small” buildings.

3.3 Surface of the building blocks

The surface of each building is represented by a quadrangle mesh. A quadrangle face belongs to the outer surface if its neighboring cell is composed of ground or air. In addition, the orientation of the quadrangle face is straightforward since the normal vector to the face must point out of the building cell which contains the face.

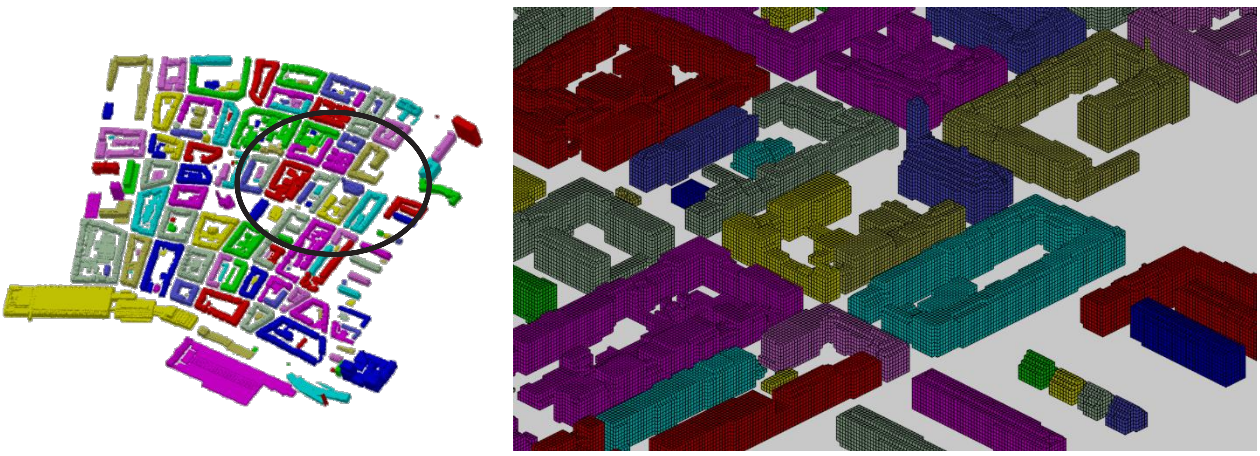
3.4 Elimination of boundary voxels

Building voxels have been assigned to cells which contain other points than ground points. We have identified the volumes and build the outer surface of each volume. The link between a quadrangle outer face and the cell which contains the face is kept. If no building point can be found at a given distance (in practice 0.6) from the outer face, the voxel which contains the face is eliminated. The idea is to keep the voxel if the volume shared with the building is sufficient. This process eliminates from the surface boundary the points which are "far" from the boundary of the building to represent. As shown in figure 3a, 4 cells which are marked by a cross are eliminated.



Figures 3. Elimination of boundary voxels

After the process, for each volume, both list of boundary voxels and outer boundary faces are updated. Volumes of different components can already been calculated. An example of voxel representation of a district of Basel is illustrated in figures 4. Each building blocks is represented. The ground is not displayed on the figure.



Figures 4. Voxel representation of a district of Basel

4 Improving the voxel model

4.1 Smoothing the voxel model

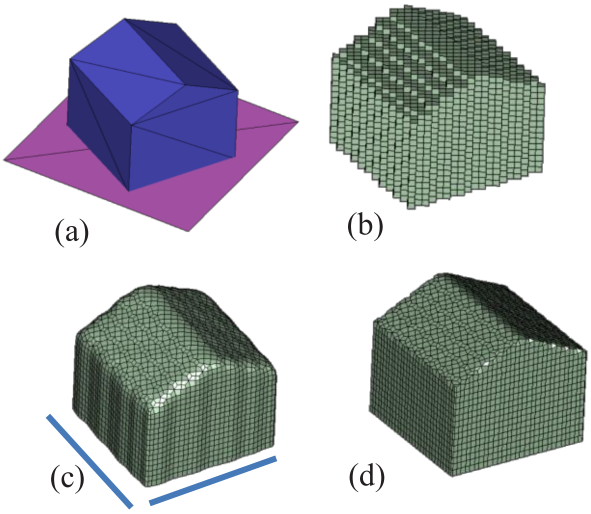
Voxelisation provides an efficient and robust way to build a topology of highly complex models while eliminating unnecessary details, noisy data. Moreover, the mesh obtained can be used for a number of simulations [Hello 2014]. The mesh at the interface between ground/building is conforming. Indeed, the accuracy of the representation is driven by the voxel size and the cost can be quickly prohibitive. In addition, the computation of the normal vector at each face as well as the nodal averaged normal vector is very poor. Many applications [Beckers 2013] require a "decent" computation of the normal vector, especially on the rooves of the buildings. A common idea is to project the points of the voxel model on the set of data points in the direction of the normal vector to the surface. The lack of accuracy of the normal vector makes this procedure quite unreliable. The idea of this work is to smooth the voxel model, not only for esthetical purposes but also to improve the computation of the normal vector at each face and node. Surface smoothing techniques are mainly used for graphics applications, geometric modeling or even film animation. Among them, subdivision techniques [Catmull 1978] are very popular because they can be easily implemented. As in laplacian smoothing, the principle is to mode a point to a location calculated by a weighted combination of its neighbors position. We have applied here a standard Catmul-Clark algorithm in which no additional faces nodes are added but only vertices are updated. In practice, the process of smoothing is repeated 3 times. For some applications, the main drawback is the shrinkage of the volume. As we plan to project the points of the voxel model, we experienced that the shrinkage does not affect the a posteriori projection procedure. The smoothing procedure is applied to the buildings and to the ground. The position of a node located at the interface ground/building and air/building is provided by the smoothing of the building only. The quality of the surface mesh is controlled at each step of the smoothing. The new location is updated only if the quality of the mesh remains acceptable.

4.2 Projection of the voxel points on the initial data

In order to project the surface voxel points on the initial data, the first step is to calculate the normal vector at each node. This is obtained by averaging the normal vector of all quadrangle faces coincident to that node. In order to compute this normal vector, quadrangles are split into 2 triangles. As two possibilities exist, the solution chosen is the one which maximizes the shape quality criterion defined as the ratio of the inscribed radius and the longest edge [Rassineux 2000]. In a number of cases, the quadrangle mesh can be twisted and only one possibility is acceptable. Then the closest point of the initial data in the direction of the normal vector is chosen. A set of neighboring data point is selected around this point in order to approximate a moving least square plane. If the normal to this plane is compatible with the average normal vector at the voxel point, the voxel point is projected on the estimated plane, otherwise another data point is chosen. The quality of the surface mesh is also controlled. The new location is updated only if the quality of the mesh remains acceptable.

5 Results

A STL mesh of a very simple building is first provided in figure 5a and its voxel representation in figure 5b. In order to test the performance of both smoothing and projection, the facade walls initially parallel to the (X,Y) axis have been rotated by 45°. The result of the smoothing is shown in figure 5c. The result of the smoothing at the intersection between roof and facades looks like fillets. Some nodes belong to the intersection between ground, the building and the air. On figure 5c, these nodes are closed to the blue lines. For these nodes, the smoothing is applied but their altitude (Z direction) is not changed and only the coordinates in directions X and Y are updated as show in figure 5c. The idea is to keep the façade perpendicular to the ground. The result of the projection is shown is figure 5d. The reconstruction is satisfactory even if angles between planes are smoothed.



Figures 5. Different steps of the methodology. (a) STL mesh (b) Voxel representation
(c) Constrained smoothing (d) Final result after projection

A reconstruction and a therefore a full remeshing of a district of Basel is presented. The initial data is a STL file presented in figure 6a. The STL file can be decomposed into sets of triangle surfaces, volumes. A building block can be the assembly of a high number of intersecting volumes and surfaces what explains the different colors for a same building block on figure 6a.The final result of the reconstruction is shown in figure 6b. 140 building blocks have been identified.

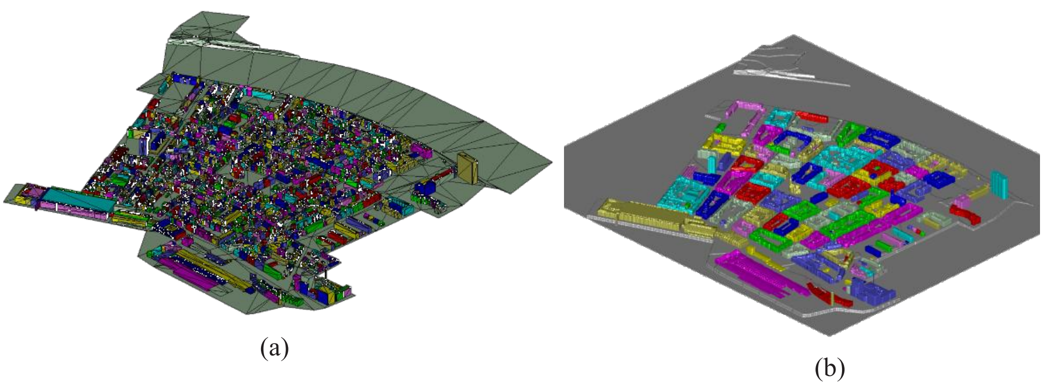


Figure 6. STL representation of a district of Basel- Full model

A more detailed view of the STL (figures 7a) and the smoothed voxel model (figures 7b) is presented thereafter. Apart from angles between planes, the main features of the original model seem to be globally well represented. A selection of complex building is displayed in figures 8. When shapes become more complex, the projection of the voxel nodes may degrade the quality of the mesh.

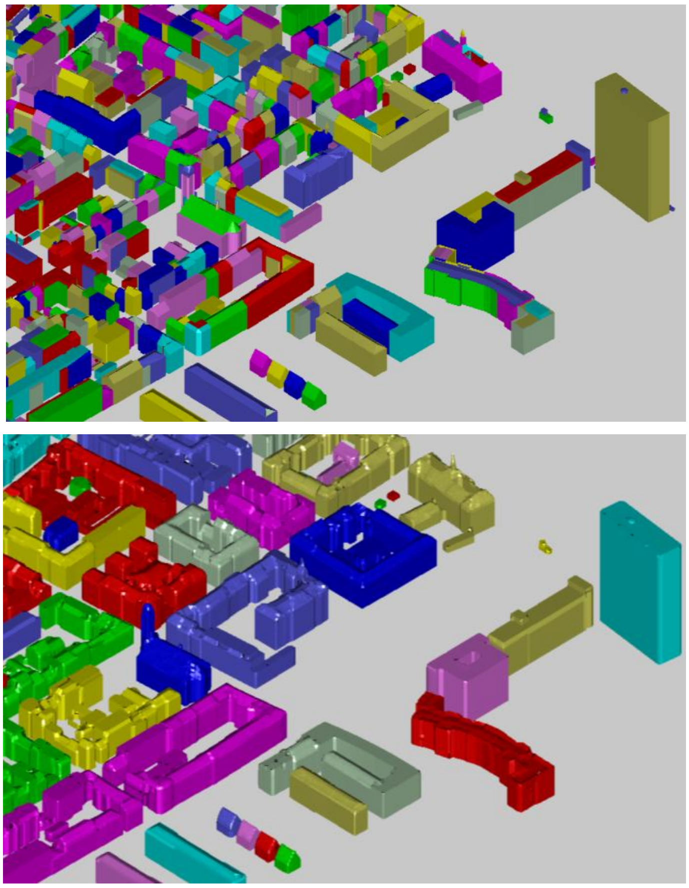
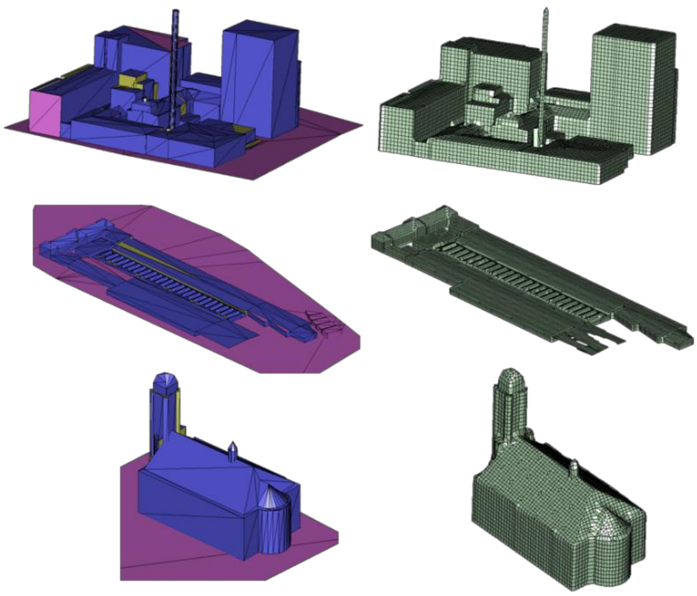


Figure 7. STL representation of a district of Basel- detailed view
(a) STL mesh. (b) rebuild model



Figures 8. Selection of complex buildings. (a) STL meshes (b) Final meshes

6 Conclusions

- A voxel based methodology to create high quality finite element meshes of large city models for urban computational physics from the data of STL files or 3D points has been presented
- The methodology cannot be used if an accurate representation of the geometry is expected. However, the main features of the models presented here are globally well rebuilt.
- The evolving voxel structure which is kept during the process guarantee that conforming meshes are created.
- The surface mesh obtained, once split into quadrangles can be coarsened with standard techniques [Rassineux 2000]
- The closed surface meshes obtained for the air, ground, building can be easily meshed thereafter by any unstructured mesh generator [Rassineux 1997].
- The smoothing procedure allows a better representation of the normal vector to the surface and therefore enables a projection on the initial data
- The process before projection is very robust. The quality of the mesh must be strictly controlled during the projection procedure.

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